SHEAR FAILURE OF LARGE LIGHTLY REINFORCED CONCRETE BEAMS: PART II – ASSESSMENT OF GLOBAL SAFETY OF RESISTANCE

Vladimir Cervenka, Cervenka Consulting, Prague, Czech Republic
Jiri Dolezel, Brno University of Technology, Czech Republic
Drahomir Novak, Brno University of Technology, Czech Republic

ABSTRACT

Global safety format is applied for verification of safety of resistance calculated by non-linear analysis for brittle model of failure. The case study is based on two experiments of large reinforced concrete beams, one without shear reinforcement and another with light shear reinforcement. Various safety formats are compared including: Global safety factor according to Eurocode 2 – for bridges; Global safety method based on estimate of coefficient of variation (ECOV method); Full probabilistic analysis; Partial safety factor method. In all methods the resistance is calculated by non-linear finite element method. Results compare design resistance obtained by various methods. The study confirms that the shear resistance of large beams can be well simulated by nonlinear analysis, while it is significantly underestimated by conventional methods.

Keywords: Global Safety, Partial Factors, Shear Failure, Nonlinear Analysis.

INTRODUCTION

Global safety format has been proposed for verification of safety of resistance calculated by non-linear analysis. It has been introduced in the Eurocode 2 for bridge structures. Non-linear analysis can be regarded as a virtual testing, where a section-oriented local safety verification is not relevant and a global safety format should be applied. The authors have been recently involved in research dealing with this topic and offered a comparison of global safety obtained by several methods Cervenka[1]. This investigation included some typical concrete structures with failure modes due to bending and combined shear-bending and compression. In these structures the ductile mode of failure was prevailing. It confirmed that simplified global safety formats can be used in practise. The present research extends the range of failure to large beams with brittle shear failure, tested by Collins and Yoshida[2], where the size effect is important and brittleness is increased.

SAFETY FORMATS

Full probabilistic analysis

In general, probabilistic analysis is the most rational tool for safety assessment of structures. It can be further refined by introducing non-linear structural analysis as a limit state function. The numerical simulation resembles a real testing of structures by considering a representattive group of samples, which can be statistically analyized for assessment of safety. We shall only briefly outline this approach. More about the probabilistic analysis is presented in other publications by authors, such as Novak et al [4].

Variability of basic properties is described by distribution functions and its parameters (mean, standard deviation, etc.). Probabilistic analysis of resistance is performed by numerical method such as LHS sampling method. Resulting array of resistance values is approximated by a distribution function of global resistance and describes random properties of resistance. Finally, for a required reliability index β, or probability of failure P_f, a value of design resistance R_d shall be calculated. The probabilistic analysis in this study is made with the help of software SARA, which integrates programs tools ATENA and FREET.

Probabilistic analysis is so far an ultimate tool for safety assessment. It can reveal reserves, which cannot be discovered by conventional methods. It is superior to simplified methods described later in this paper because it can provide a variability of resistance specifically for each case. It is well known that this variability depends on type of failure, reinforcing, etc., see Holicky [5] and thus assuming a unique global safety factor is theoretically questionable. However, full probabilistic analysis is computationally demanding and requires good information about random properties of input variables. Therefore, it is applied mainly in special cases, where consequences of failure substantiate a full reliability analysis.

Note, that we are dealing with the random variation of resistance only. Thus, we keep the safety separated, with probability of resistance on one side and probability of design load on the other. This makes the verification of safety in analogy with the current partial factors more convenient.

ECOV method – estimate of coefficient of variation

This method was inspired by the global safety analysis presented by Holicky [5]. It is based on the idea, that the random distribution of resistance, which is described by the coefficient of variation V_R, can be estimated from mean R_m and characteristic values R_k of resistance. The underlying assumption is that random distribution of resistance is according to lognormal distribution, which is typical for structural resistance. In this case, it is possible to express the coefficient of variation as:

$$V_R = \frac{1}{1.65}\ln\left(\frac{R_m}{R_k}\right) \tag{1}$$

The global safety factor γ_R of resistance is then estimated as:

$$\gamma_R = \exp(\alpha_R \beta V_R) \tag{2}$$

where α_R is the sensitivity factor for resistance (as defined by FORM) and β is the reliability index. The above procedure enables to estimate the safety of resistance in a rational way, based on the principles of reliability accepted by the codes. Appropriate code provisions can be used to identify these parameters. According to Eurocode EN 1990, typical values are $\beta = 3.8$ (50 years) and $\alpha_R = 0.8$. It correspond to the failure probability P_f =0.001. The global resistance factor is then:

$$\gamma_R \cong \exp(3.04\, V_R) \tag{3}$$

and the design resistance is calculated as:

$$R_d = R_m / \gamma_R \tag{4}$$

The keystone in this method is a determination of the mean and characteristic values of resistance R_m, R_k. It is proposed to estimate them by two separate nonlinear analyses using mean and characteristic values of input material parameters, respectively.

It can be argued, why not to calculate R_d directly from Eq.(2) as we do in partial factor method. One of the reasons is the fact that design values f_d are extremely low and do not represent a real material. A simulation of real behaviour should be based on mean material properties and safety provision should be referred to it. Analysis based on extremely low material properties may result in unrealistic redistribution of forces, which may not be on the conservative side. It may also change the failure mode. Characteristic value f_k is not so far from mean and well reflects a scatter of resistance.

The method is general and reliability level β and distribution type can be changed if required. It reflects all types of failure. The sensitivity to random variation of all material parameters is automatically included. Thus, there is no need of special modifications of concrete properties in order to compensate for greater random variation of certain properties. However, the method requires two separate non-linear analyses.

Method based on EN1992-2

Eurocode 2 for bridges introduced a concept for global safety verification based on nonlinear analysis. Design resistance is calculated from

$$R_d = R(f_{ym}, \tilde{f}_{cm} ...) / \gamma_R \tag{5}$$

Where $f_{ym}, \tilde{f}_{cm}$ are mean values of material parameters of steel reinforcement and concrete, $f_{ym} = 1.1\, f_{yk}$ and $\tilde{f}_{cm} = 0.843\, f_{ck}$. Note that concrete mean value is reduced to account for higher variability of concrete property. For more details see Bertagnoli et al.[3]. The global factor of resistance shall be $\gamma_R = 1{,}27$. The evaluation of resistance function is done by nonlinear analysis assuming the material parameters according to the above rules.

Partial safety factors (PSF)

Method of partial safety factors, which is used in most design codes, will be described briefly. In this method the design condition is formulated as

$$E_d < R_d \tag{6}$$

Design action $E_d = E(F, \gamma_G, \gamma_Q, \gamma_P, ..)$ is function of representative load F, which is factorized by partial safety factors $\gamma_G, \gamma_Q, \gamma_P, ..$ for permanent load, live load, prestressing, etc. Resistance $R_d = R(f_d)$ is based on design values of material parameters $f_d = f_k / \gamma_M$, where f_k are characteristic values and γ_M partial safety factors of materials.

Verification of safety by condition (6) in the present design practice is applied to cross sections and actions are obtained by linear analysis. It is well known that this concept is not consistent, since different methods are used for actions (linear analysis) on one side, and for resistance of cross sections (nonlinear) on the other. Furthermore, only local safety check is exercised and a global safety assessment is not performed and unknown.

The approach of partial factors can be simply extended by applying nonlinear analysis in order to overcome the above mentioned deficiencies. Then, the action E_d in condition (1) is considered on global level (for example live load intensity) and resistance R_d is an ultimate load intensity obtained by nonlinear analysis, in which design values of material parameters f_d are used. One of the aims of this study is to verify the global safety of the resistance when using PSF method in nonlinear analysis.

Numerical analysis

Examples in this paper are analysed with program ATENA for non-linear analysis of concrete structures. ATENA is capable of a realistic simulation of concrete behaviour in the entire loading range with ductile as well as brittle failure modes as shown in papers by Cervenka [1]. It is based on the finite element method and non-linear material models for concrete, reinforcement and their interaction. Tensile behavior of concrete is described by smeared cracks, crack band and fracture energy, compressive behavior of concrete by a plasticity model with hardening and softening. The constitutive model is described in detail by Cervenka [6]. In the presented examples the reinforcement is modelled by truss elements embeded in two-dimensional isoparametric concrete elements. Nonlinear solution is performed incrementally with equlibrium iterations in each load step.

LARGE LIGHTLY REINFORCED BEAMS

In this study the large beams tested at the University of Toronto by Collins and Yoshida [2] were investigated numerically. A good knowledge of material parameters is essential for such a task. This was a subject of an accompanying study by Lehky et al.[7] on identification of material parameters by inverse analysis. The objective of this study is the safety verification based on global approach with the aim to offer a guidance for design based on nonlinear analysis. In earlier studies authors have presented results of global safety formats for some typical structures of usual sizes and ductile failure types. The present study deals with large beams failing in shear an exhibiting highly brittle failure.

Two beams from the experimental program of Yoshida are considered: Beam YB2000/0 without shear reinforcement and beam YB2000/4 with vertical reinforcement by 8 T-headed bars. The beams are schematically depicted in Figure1 and 2.

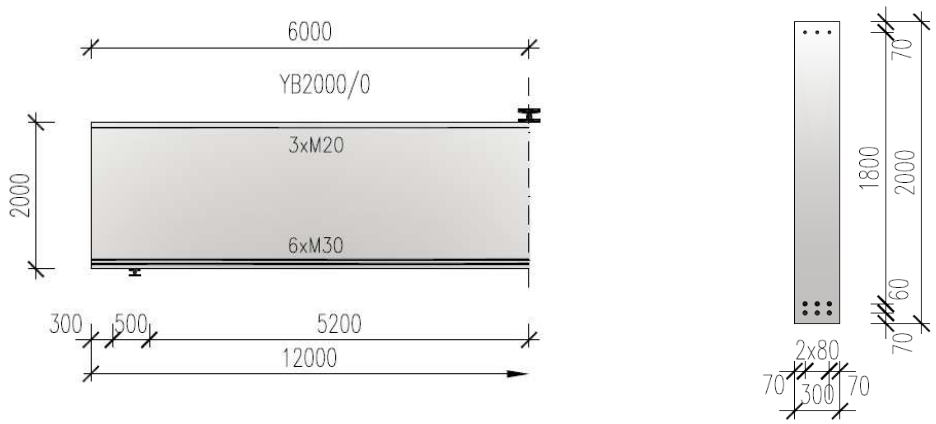


Figure 1 Beam YB2000/0 dimensions and reinforcement.

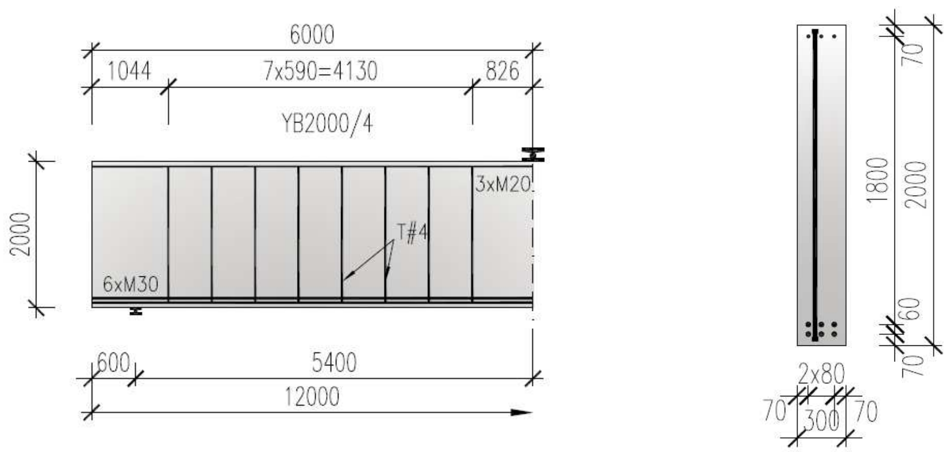


Figure 2. Beam YB2000/4 dimensions and reinforcement.

The longitudinal reinforcement in both beams was identical. The reinforcing ratio of bottom reinforcement by 6xM30 bars was 0.0074. The ratio of vertical reinforcement by T-headed bars T#4, spacing 0.59m was 0.00071. The beams were only lightly reinforced. The shear span ratio a/d=2.86 indicates a shear critical geometry.

The experimental study[2] offered for concrete property only a compressive strength at the date of testing, which was obtained from cylinder tests. In the tests slightly different properties were found in two specimens. However, in this study it was decided to use identical concrete properties in both specimens in order to keep the effect of different shear reinforcing not influenced by other parameters. Thus the exact parameters resulting from the identification study[7] were not used here since the prime objective of this study was to compare various formats of global safety verification, rather then to exactly simulate experiments The assumed set of parameters for concrete and reinforcement is shown in Table 1.

Table 1 Material properties of beams.

Concrete property	Value
Elastic modulus E_c MPa	34 000
Compressive strength f_c MPa	37
Tensile strength f_t MPa	2.8
Specific fracture energy G_f N/m	80
Poisson ratio μ -	0.2
Plastic strain at f_c (peak) ε_{cp} -	0.002
Plastic end displacement w_d m	0.05

Steel property	Value
Elastic modulus E_s MPa	200 000
Yield stress f_y MPa	470
Max. stress $f_{s,,max}$ MPa	680

The fracture-plastic constitutive model in ATENA (described by Cervenka [6]) was used for concrete and the multi-linear constitutive law was used for reinforcement.

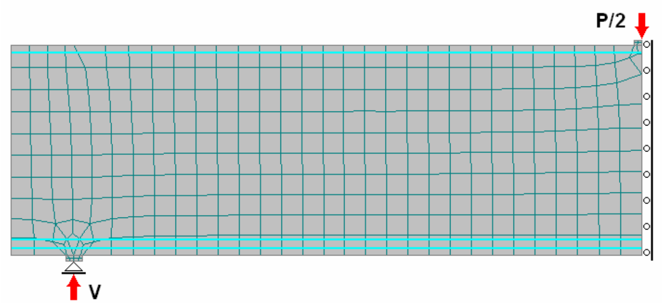


Figure 3. Finite element model.

The finite element analysis was done for a symmetrical half of the beam in plane stress representation. Quadrilateral 4-node isoparametric elements, size 200 mm, were used for concrete and embedded truss elements for bars as shown in Figure 3. The total load $P = 2V$ acting in the top centre of the beam is considered as a global resistance. Like in experiment, self weight is considered in analysis but not included in the monitored load P.

GLOBAL SAFETY ANALYSIS

Full probabilistic analysis (FP)

In probabilistic analysis material parameters were considered as random variables described by type of probability distribution function (PDF), mean and coefficient of variation (V). The parameters are summarized in Table 2.

Table 2 Random variables of concrete properties.

i	Variable			Type of PDF	mean	V
1	Elastic modulus	E_c	GPa	Lognormal 2	34	0.15
2	Tensile strength	f_{ct}	MPa	Lognormal 2	2.80	0.25
3	Compressive str.	f_c	MPa	Lognormal 2	37	0.10
4	Gracture energy	G_f	N/m	Lognormal 2	80	0.25
5	Plastic strain	ε_{psc}	-	Normal	0.002	0.10

Other parameters not mentioned in Table 2 and parameters of steel reinforcement are considered as deterministic, constant in all samples. Values of mean and design resistance, coefficients of variation and safety factors with respect to mean, are shown in Table 2.

Table 3 Results of full probabilistic analysis.

Result	YB2000/0	YB2000/4
Mean resistance P_m kN	419.2	1317,6
Coefficient of variation V -	0.189	0.117
γ_R -	1.96	1.44
P_d^{FP} kN	**213.8**	**913.3**

Results of all samples produced by SARA studio in form of load-displacement diagrams for two beams are shown in Figures 4 and 5. Experimental diagrams are also included.

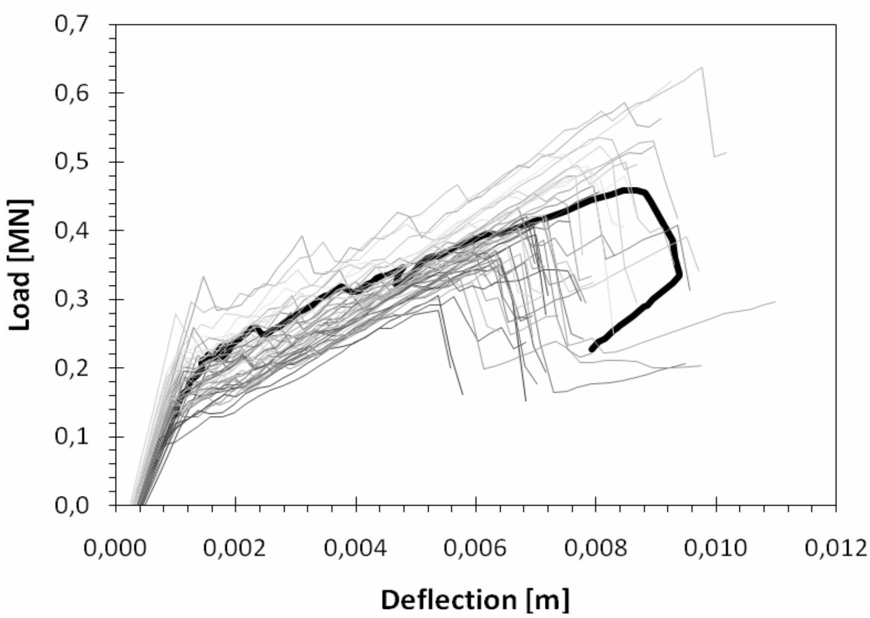


Figure 4. Beam YB2000/0. Load-displacement diagrams of 50 simulations and experiment.

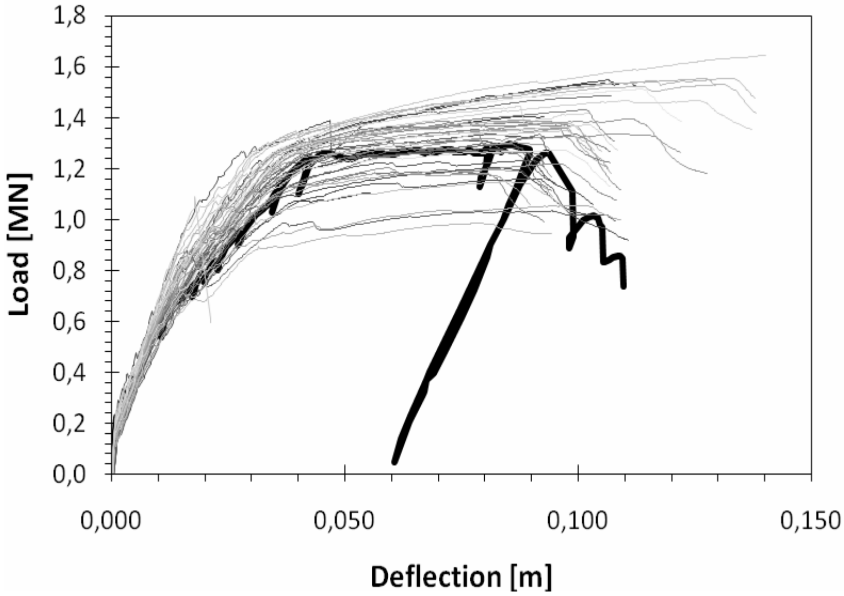


Figure 5. Beam YB2000/4. Load-displacement diagrams of 50 simulations and experiment.

ECOV method

Input variables and results of design resistances are shown in Table 4.

Tabble 4 Variables and results of ECoV method.

Variable	Mean	Charact.
Compressive strength f_c MPa	37	29
Tensile strength f_t MPa	2.8	1.96
Fracture energy G_f N/m	80	55

Result	YB2000/0	YB2000/4
P_m kN	425.8	1141.0
P_k kN	303.6	1056,0
CoV -	0,205	0,046
γ_R -	1.86	1,15
P_d^{ECoV} kN	**228.3**	**947.5**

EN1992-2 (Bridges)

Mean values of parameters for Eq. (5) are shown in Table 5. The values of tensile parameters f_{tm} and G_{fm} are calculated by the same methods as for the compressive strength $\tilde{f}_{cm}$. Mean and design resistances are in Table 5.

Table 5 Variables and results of.EN1992-2 Method.

Variable	Mean
Compressive strength $\tilde{f}_{cm}$ MPa	26.4
Tensile strength f_{tm} MPa	1.53
Fracture energy G_{fm} N/m	43.7

Result	YB2000/0	YB2000/4
P_m kN	425.8	955.2
γ_R -	1.27	1.27
$P_d^{EN1992\text{-}2}$ kN	**227.2**	**752.1**

Partial safety factors (PSF)

Design values of parameters considered for analysis and resulting design values of resistance are shown in Table 6.

Tabble 6 Variables and results of PSF Method.

Variable	Design
Compressive strength f_{cd} MPa	19.3
Tensile strength f_{td} MPa	1.21
Fracture energy G_{fd} N/m	34.5

Result	YB2000/0	YB2000/4
P_d^{PSF} kN	**213.8**	**744.4**

Comparison of design resistances

Design resistances obtained by all methods are summarized in Table 7, where relative deviations of approximate methods with respect to full probabilistic method are described by ratios P_d / P_d^{FP}. Safety of design resistance offered by various methods is compared in terms of PDF representation as shown in Fig.6 and 7.

Table 7 Summary of design resistances.

Method	**YB2000/0**		**YB2000/4**	
	P_d	P_d / P_d^{FP}	P_d	P_d / P_d^{FP}
FP	**230**	**1.00**	**913**	**1.00**
ECOV	**228**	**0.99**	**947**	**1.04**
EN1992-2	**227**	**0.98**	**752**	**0.82**
PSF	**214**	**0.93**	**744**	**0.81**
EN1992-1	**501**	**2.18**	**862**	**0.94**
Experiment	**463**	**2.01**	**1262**	**1.38**

The comparison shows that the approximate methods of global safety verification, namely the PSF method, give acceptable results at least for pure shear failure of the beam YB2000/0. The results of the other beam YB2000/4 are not so convincing and should be further investigated.

The failure mode of beams observed in numerical simulations and experiments is compared in Figure 8. The failure of beam YB2000/0 without shear reinforcement was clearly due to formation of inclined cracks followed by splitting bond failure of bottom reinforcement. Yielding of longitudinal reinforcement was not reached. This was observed in both, experiment and analysis.

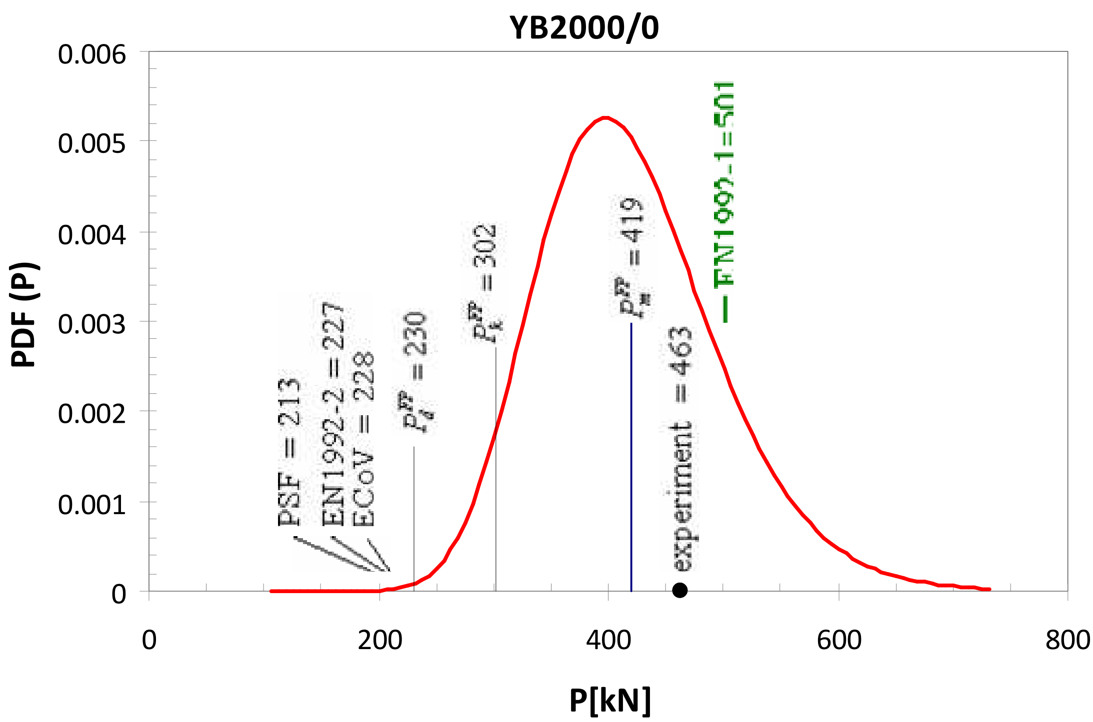


Figure 6. Comparison of safety margins for Beam YB2000/0.

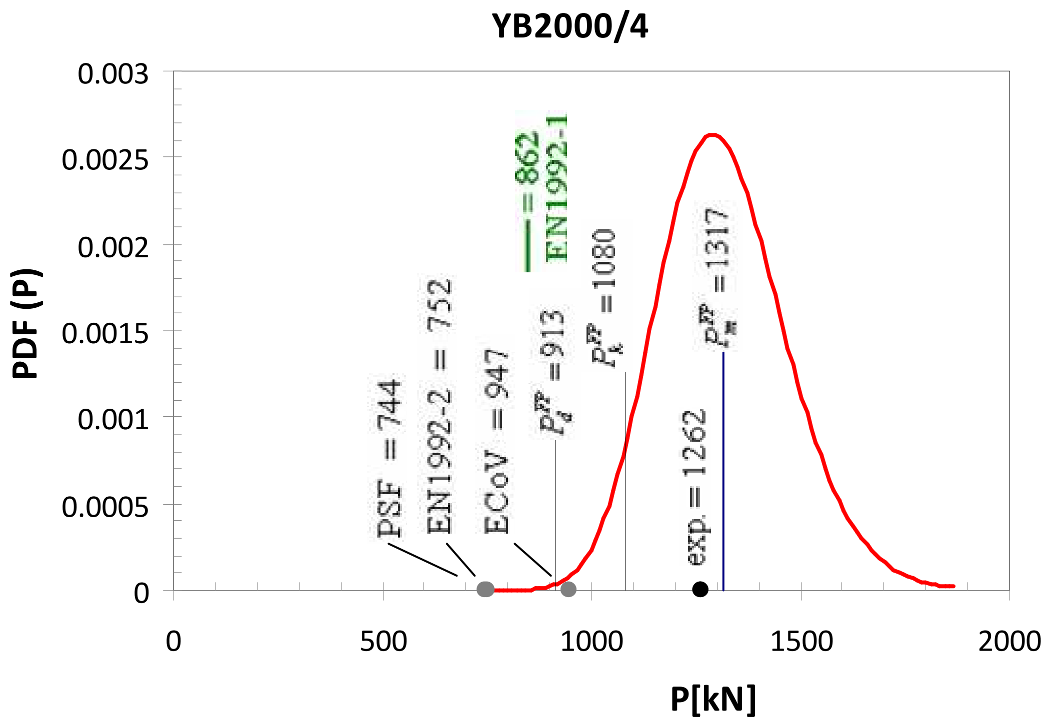
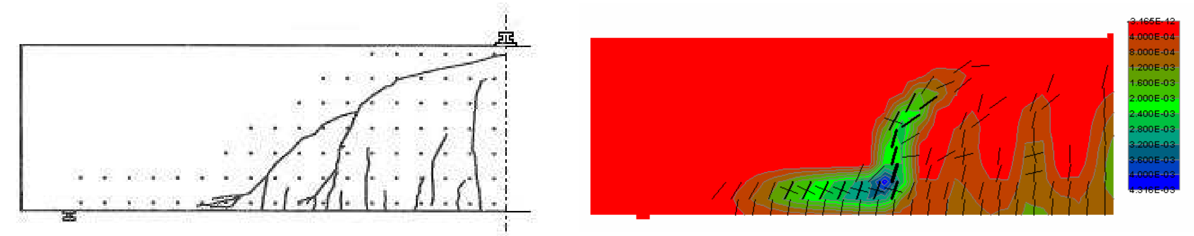
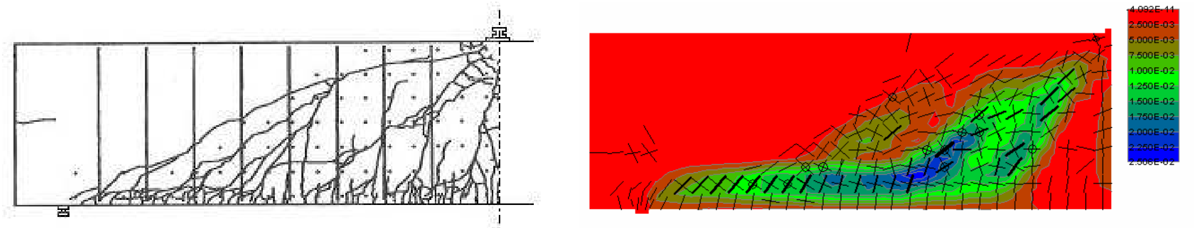


Figure 7. Comparison of safety margins for Beam YB2000/4.

In beam YB2000/4 adding a minimum amount of shear reinforcement caused an increase of shear resistance by factor 2.7. The process of failure was marked by a full exploitation of plastic capacity of vertical reinforcement during diagonal crack opening, up to the maximum stress. Longitudinal reinforcement reached yield stress but did not developed full plastic deformation. The failure was due to compressive failure of cracked concrete near the load application at the top and longitudinal splitting along the bottom bars. Thus the failure can be attributed to concrete after plastic extension of vertical T-headed bars. A relative high coefficient of variation of resistance V=0.17 of the beam YB2000/4 can be explained by high contribution of concrete to the failure.



Experiment YB2000/0 Mean simulation



Experiment YB2000/4 Mean simulation

Fig.8 Failure modes. Analysis: cracks and iso-lines of fracture strains indicating damage.

For completeness, in order to illustrate the practical consequences of this study, design values according to the Eurocode EN1992-1 are also included in Table 7. The shear resistance was calculated as minimum value required by the clauses 6.2.2 and 6.2.3. (The code values are reduced by 48 kN which is load due to self weight. This is a provision to maintain comparable values with experiment and analysis in this study.) The results indicate an overestimation of shear resistance without shear reinforcement by present code and confirm, that the size effect of concrete structures is not well captured and the required safety is not adequate. Shear resistances obtained by experiments are also included. Obviously, they are not directly comparable with design values and correspond to the mean resistances. The ratio P_{EXP} / P_d^{FP} indicate a global safety factor with respect to a real resistance represented by experiment.

Constitutive model used here is based on the smeared crack approach, Cervenka[1] , and crack band model based on fracture mechanics, Bazant[8,9], which are implemented in ATENA. According to this theory large structures are more brittle, which leads to a smaller nominal resistance.

Lack of safe and rational design of lightly reinforced large concrete structures in current codes of practise motivated the group of Collins in Toronto in pursuing systematic experimental research in this field. They developed “Modified Compression Field Theory” (MCFT) which takes into account the size effect, Bentz et al.[10]. Resistances obtained by MCFT for the beams analyzed in this study reported by Yoshida[2] are similar to those obtained by the nonlinear analysis in this study.

CONCLUSIONS

Full probabilistic analysis based on random sampling in combination with the non-linear finite element simulation is a powerful tool for safety assessment of brittle modes of failure. The study revealed that the random variability of resistance in case of brittle failure due to concrete is much larger compared to ductile modes of failure. Coefficients of variation of beams with and without shear reinforcement were found as V=0.19 and 0.11, respectively. Consequently the global safety factors of resistance for the required safety 0.001 were, γ_R=1.86 (without shear reinforcement) and 1.44 (with shear reinforcement). Only minimum amount of shear reinforcement caused a substantial (threefold) increase of resistance, a shift to a more ductile response and resulted in reduction of the coefficient of variation of resistance and of required safety margin.

Simplified global safety formats, by methods ECoV, EN1992-2, PSF, captured well the case of shear failure without shear reinforcement and indicated, that they can be used for design based on non-linear analysis for brittle models of failure.

The beam with light shear reinforcement indicated more diverse results. Simplified methods EN1992-2 and PSF underestimated the design load, which is acceptable for practical application. Method ECoV underestimated the coefficient of variation of resistance giving slightly greater design load as compared with full probabilistic method.

All methods presented in this study gave better and safer design resistances for beam without shear reinforcement then Eurocode EN1992-1.

ACKNOWLEDGEMENT

This research work was financially supported by the Czech Grant Agency project 103/08/1527. The authors are indebted to M.P.Collins, University of Toronto, who provided experimental data for the study.

REFGERENCES

1. Cervenka V., *"Computer simulation of failure of concrete structures for practice"*, The First fib Congress - Concrete structures in the 21st century, *Osaka, Japan 2002.*

2. Yoshida, Y. *"Shear reinforcement for large lightly reinforced concrete members"*, MS thesis, Dep. Civil Engineering, University of Toronto, 2000.

3. Bertagnoli, G., Giordano L., Mancini, G. (2004). *"Safety format for the nonlinear analysis of concrete structures."* STUDIES AND RESEARCHES –V.25, Politech. di Milano, Italy.

4. Novák, D., Vořechovský, M., Lehký, D. & Rusina, R., Pukl, R. & Červenka, V. *„Stochastic nonlinear fracture mechanics finite element analysis of concrete structures.*" Proceedings of 9th Int. conf. on Structural Safety and Reliability Icossar, Rome, Italy, 2005.

5. Holický, M. *„Global resistance factors for reinforced concrete members"*. ACTA POLYTECHNICA 2006, CTU in Prague.

6. Cervenka, J., Pappanikolaou, V. *"Three dimensional combined fracture-plastic material model for concrete"*. Int. J. of Plasticity, Vol. 24, 12, 2008, ISSN 0749-6419, pp. 2192-2220.

7**.** Lehky, D., Novak, D., Cervenka, V. *"Shear Failure of Large Concrete Beams: Part I – Material Parameter Identification"* . 3rd *fib* Int. Congress, Washington, 2010.

8. Bazant, Z.P., Oh, B.H. "*Crack band theory for fracture of concrete"* Materials and Structures, RILEM 16, (3), 1983, 155–177.

9. Bazant, Z.P., Planas, J. "*Fracture and size effect in concrete and other quasibrittle materials*", CRC Press, Boca Raton, Florida, 1998.

10. Bentz, E.C., Vecchio, F.J., Collins, M.P. "*Simplified Modified Compression Field Theory for Calculating Shear Strength of Reinforced Concrete Elements*". ACI Material Journal, Jul/Aug 2006.