

MODEL UNCERTAINTY IN NONLINEAR ANALYSIS FOR DIGITAL TWINS FOR THE ASSESSMENT OF NUCLEAR CONTAINMENTS

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ABSTRACT

Nonlinear numerical simulation is emerging as a robust alternative or supplementary method for design verification as well as assessment of existing structures, particularly in the nuclear power plant (NUPP) sector. It is widely utilized during the design phase and in service extension processes. This paper discusses the treatment of model uncertainty in nonlinear finite element modelling of concrete structures and validation using blind predictions. The objective is to utilize nonlinear analysis in a digital twin framework for controlled aging management of an existing nuclear containment structure. ATENA finite element nonlinear modelling software is used to simulate the structural response to mechanical as well as non-mechanical loading. The model response is calibrated using the data from temperature as well as strain sensors. The analysis incorporates advanced fracture-plastic material models and considers the influence of prestressing tendons as well as normal reinforcement. The paper presents the initial results of the ongoing project. The findings contribute to the improvement of structural assessments for nuclear power plants, supporting the development of digital twins for long-term operational monitoring and safety evaluations.

INTRODUCTION

The application of the finite element method for nonlinear analysis of reinforced concrete structures was introduced in the 1970s through important works by Ngo and Scordelis (1967), Rashid (1968), and Červenka V. and Gerstle (1971). Initially, material models were based on the finite element method, where the specific material behavior was formulated as a constitutive model applied at each integration point to evaluate internal forces. However, De Borst (1986) and Rots (1989) soon recognized that matrix models with strain softening, if not properly formulated, exhibit strong mesh dependency and tend to approach zero energy dissipation as the element size decreases described by Bažant (1976).

To address the issue of energy dissipation converging to zero, Bažant and Oh (1983) introduced the crack band approach. More rigorous solutions to the ill-posed problem of strain softening include nonlocal models or higher-order continuum models, which incorporate additional material parameters related to the internal length scale. Although mathematically robust, these models are rarely used in engineering practice or implemented in commercial finite element codes. The limitations of the crack band model in practical engineering calculations, particularly for large finite elements or reinforced structures, were already recognized by Bažant and Oh (1983). Červenka (2018) further analyzed these limitations and proposed modifications for nonlinear finite element analyses involving both large and small elements.

Growing concerns about the safety and sustainability of nuclear power plants have significantly influenced the maintenance of existing structures. Over their lifespan, extensive research efforts have introduced various innovations in structural assessment and design. A novel approach is the concept of digital twins when a virtual model is developed, calibrated as well as gradually updated based on the data obtained from sensors attached or inserted into the real structure. The computational material model used in this paper was previously developed and validated by Červenka (1998, 2008). The applications in

nuclear industry are described in Červenka et al. (2011) or Proske et al. (2013). This study mainly focuses on structural deformations induced by change on thermal loading during the operational NPP shutdown.

NONLINEAR ANALYSIS AND CRACK BAND MODEL

The calculations in this paper were done using the finite element software ATENA (Červenka 2025). The conclusions and recommendations regarding the treatment of model uncertainty are therefore specific to this software or, at most, to models based on the smeared crack approach and crack band method. The material model is a fracture-plastic model, detailed in Červenka (1998) and Červenka and Papanikolaou (2008). The Menetrey and Willam (1995) model is employed to describe concrete plasticity under multiaxial compression (Figure 1-a) with nonlinear hardening and softening behavior (Figure 1-b, c).

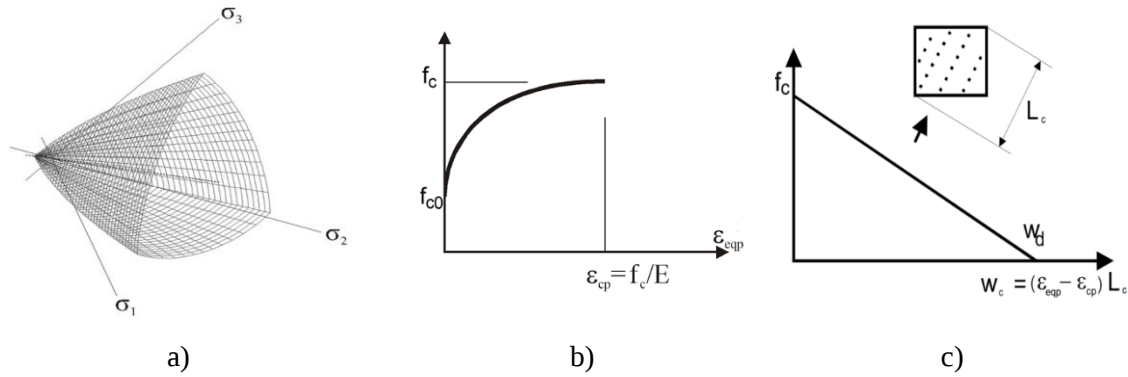


Figure 1. Three-parameter Menetrey and Willam (1995) concrete failure criterion in principal stress frame (a), hardening/softening law for plasticity model of concrete under compression (b, c).

For tensile cracking, the Rankine criterion with exponential softening, as proposed by Hordijk (1991), is utilized (Figure 2 – left image), where w_t represents the crack width. The crack band model of Bažant and Oh (1983) is applied to link crack opening displacement to fracturing strains. A similar approach is adopted for compression following Červenka et al. (2014). The crack band L_t and crush band sizes L_c are adjusted based on the crack orientation method, where the crack angle θ is taken as the average angle between the crack direction and element edges (Figure 2 – right image).

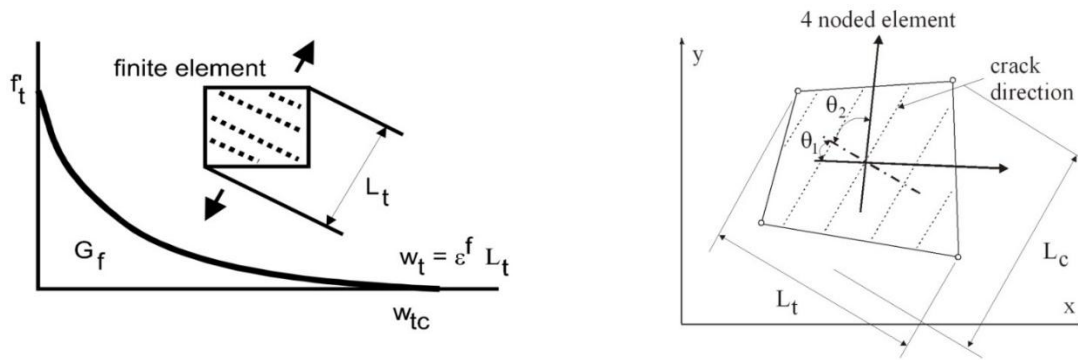


Figure 2. Rankine criterion with exponential softening of Hordijk (1991) during tensile cracking (left image). Crack band formula (right image).

MODEL UNCERTAINTY

Model uncertainty in resistance, based on nonlinear finite element analysis (NLFEA), is important for ensuring reliable design applications. It is determined by comparing model predictions

with experimental data, with the partial factors of uncertainty being valid only for the specific material model or simulation software used

The evaluation of model uncertainty, based on a database of 33 reinforced concrete members with various failure modes, is described in greater detail by Červenka V. et al (2018). This study utilized the ATENA software (Červenka V. et al 2025) with the fracture-plastic concrete material model described by Červenka J. et al (1998, 2008). The resulting model uncertainty parameters are summarized in Table 1. These uncertainty factors should be incorporated into any NLFEA applications in engineering practice and used to adjust the structural or member resistances calculated through numerical simulation.

Table 1. Patial safety model factor for model uncertainty by Červenka V. et al (2018).

Failure type	μ_0	V_0	γ_{Rd}
Punching	0.971	0.076	1.16
Shear	0.984	0.067	1.13
Bending	1.072	0.052	1.01
All failure modes	0.979	0.081	1.16

BLIND COMPETITIONS

Verification of simulation models for concrete structures is typically conducted through comparison with experimental data. Interesting insight can be obtained from blind predictions in international competitions, when the experimental results are not known at the time of the analysis. Figure 3 summarizes seven such contests in which the authors participated. The predicted strength is normalized by the ratio F_{sim}/F_{exp} with 22 cases from seven benchmark contests displayed on the horizontal axis. The vertical bars represent the prediction scatter, while the author's results are marked in green.

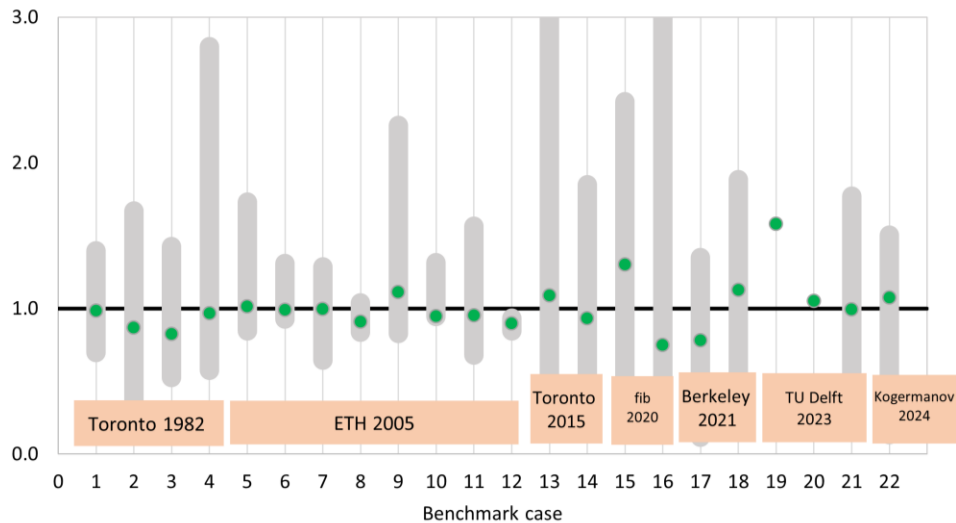


Figure 3. Benchmark summary.

Over the past 40 years, the engineering community has shown sustained interest in improving simulation tools; however, no clear trend toward reduced uncertainty has emerged. The benchmarks primarily focus on shear or bending strength, and the wide prediction scatter reflects a limited understanding of shear failure. In addition, strength, stiffness, deformations, and crack patterns were also considered in the evaluation.

APPLICATIONS IN NUCLEAR INDUSTRY

The main object of this paper is to apply nonlinear modelling in the concept of digital twins. The important part of the process is the calibration, identification of model parameters and their update by existing sensor data. This section provides a detailed description of the geometrical and material models used to calculate temperature distribution during the normal operation and regular operational shutdown. Then, the computed thermal strains are used to evaluate the deformations, strains and stresses.

In the numerical model, the containment structure is divided into two main components. The first component comprises the substructure, including the walls and slabs beneath the containment, while the second component represents the containment itself. The geometric representation of the structure is illustrated in Figure 4. The construction is reinforced with prestressed cables arranged in a highly complex shape corresponding to the reality. The routing and geometry of the prestressing cables for both the wall and the dome can be seen in Figure 5.

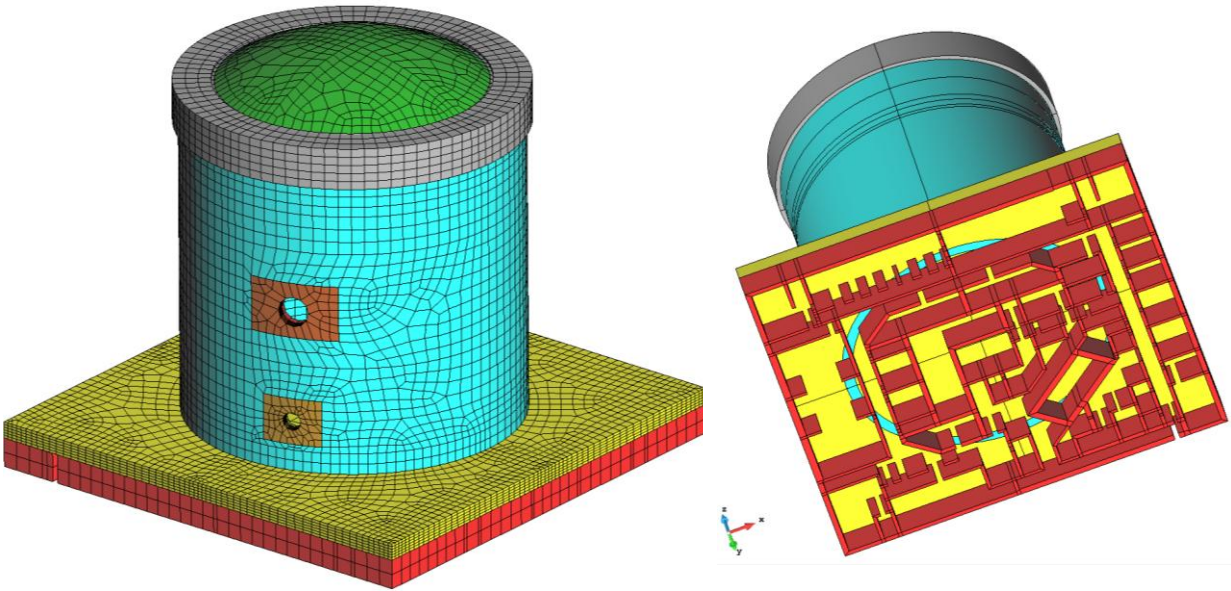


Figure 4. Finite element mesh of the numerical model using hexahedral elements on the left, and an illustration of the walls beneath the structure used as support models on the right.

The calculation was performed using ATENA 2025 software. The Transport module was utilized to simulate heat transfer, computing the temperature distribution based on the prescribed boundary conditions applied to the structure's surfaces. The resulting temperature strains can subsequently be incorporated as a non-mechanical load in the ATENA Static module.

The calculation considers only the concrete structure, for which the suitable material model was developed. This mode requires the calibration and inverse analysis for the determination of three key parameters. The first parameter is the initial temperature ($T_{init,mature}$) set at 22 °C. The other two parameters entered to the calculation are heat capacity ($c_{c,mature} = 2.55 \text{ MJ/m}^3/\text{°C}$) and thermal conductivity ($K_{c,mature} = 2.1 \text{ W/°C/m}$).

The model was divided into four zones (Figure 6.), where each was subjected to distinct boundary conditions. These boundary conditions were derived from the temperature measurements conducted in 2024 during a planned operational shutdown. To ensure accuracy, an optimization process was performed on the measured data to establish temperature functions and the temperature coefficient of heat transfer for each zone (U_i). The optimization methods used to determine the values are described in Kučerová A. et al (2023). The temperature functions applied as boundary conditions in the calculations are shown in Figure 7.

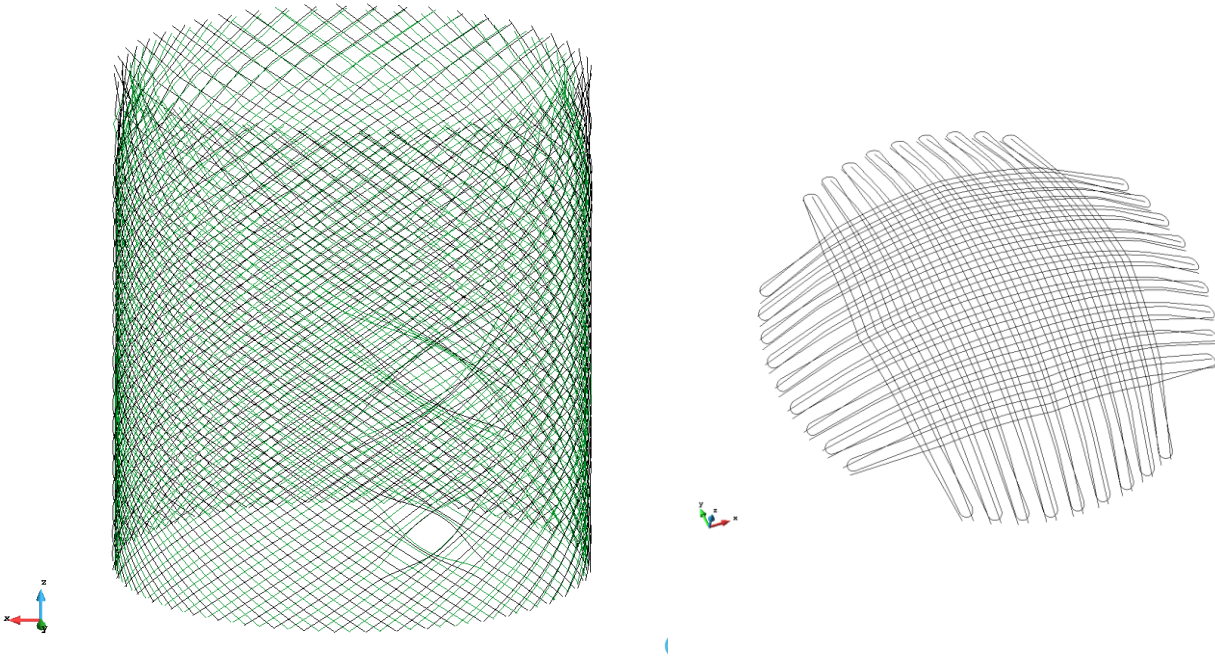


Figure 5. Prestressed cables used in numerical model.

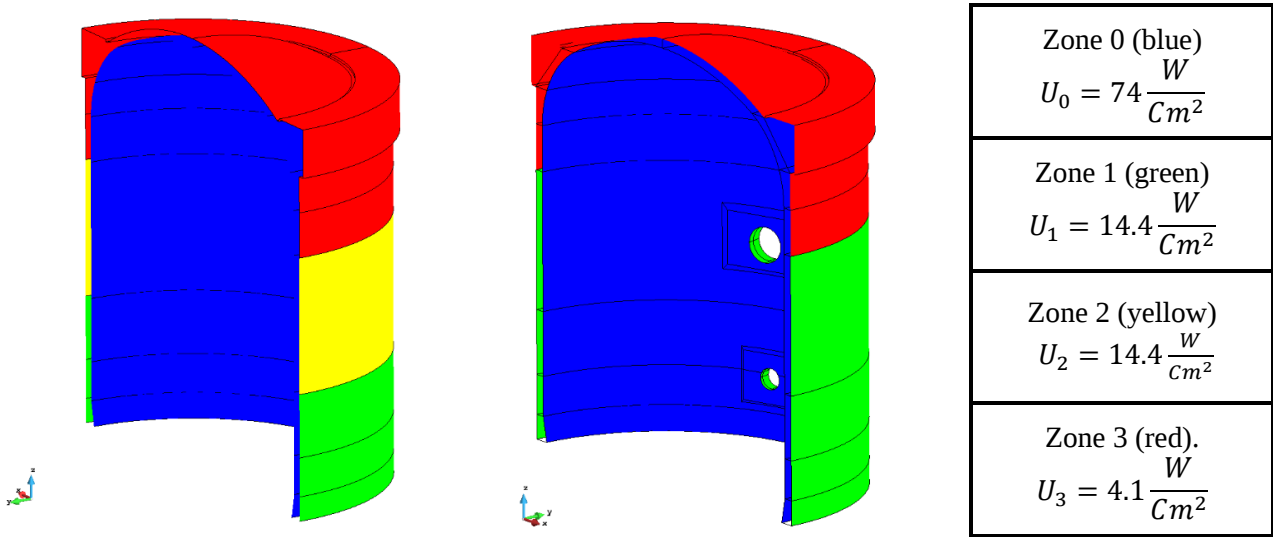


Figure 6. Illustration of the model zones with corresponding heat transfer coefficients used in the Transport calculation: Zone 0 – interior, Zone 1 – cold rooms, Zone 2 – heated rooms, and Zone 3 – outer environment.

To validate the accuracy of the calculation, temperature monitoring points were incorporated into the model to track temperatures on the inner and outer surfaces, as well as at the middle of the wall thickness. These monitoring points were strategically placed at locations corresponding to the actual sensor positions within the structure. A comparison between the calculated results and the measured temperature data is presented in Figure 8.

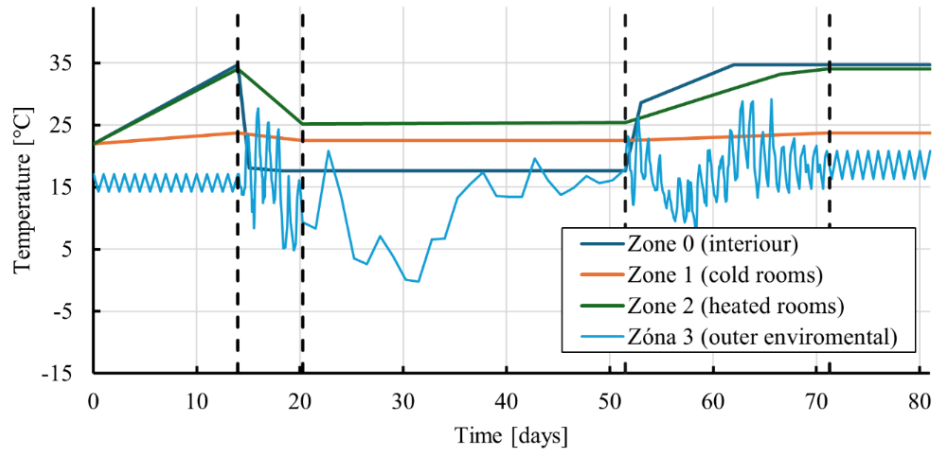


Figure 7. Boundary condition of the temperature functions used in the calculation to determine the temperature strain due to nonlinear heat transfer.

The results from the nonlinear heat transfer analysis were applied as a non-mechanical load to evaluate the tensile strain response in concrete. The mechanical properties of the concrete material model correspond to class C30/37. The model incorporates smeared reinforcement, which is defined by the corresponding reinforcement ratios and the directional vector. The main material parameters used in the calculation are summarized in Table 2.

Table 2. Characteristic material parameters for concrete material (corresponding to the concrete class C30/37), prestressed cabled and smeared reinforcement used in the Static Module.

Parameter	Value	
Concrete C30/37		
Elastic modulus E [GPa]	32	
Poisson ratio ν [-]	0,2	
Compressive strength f_c [MPa]	-30	
Tensile strength f_{ct} [MPa]	2	
Fracture energy G_f [N/m]	50	
Coefficient of heat expansion α [K ⁻¹]	0,000012	
Reinforcing	Prestressed cables	Smeared r.
Cable diameter d [mm]	170	-
Reinforcing ratio ρ [-]	-	0,0027 / 0,00409 / 0,0013
Elastic modulus E [GPa]	200	210
Yield strength f_{yk} [MPa]	1620	445
Tensile strength f_{ytK} [MPa]	1800	490
Limit strain ε_u [-]	0,04	0,05

Monitors were implemented in multiple directions to track the tensile strain in the concrete structure. These monitors were positioned in the model at locations corresponding to their real-world counterparts within the construction. Figure 9. illustrates the placement of all monitors used in the model.

Calculation shows increase in tensile strain in the radial and vertical (interior) direction monitor at the start of cooling the structure at the beginning of the operational shutdown. The tensile strain value

subsequently decreases but does not reach zero value, indicating a residual tensile stress in the structure. The comparison of the calculated and measured strains is show in Figure 10.

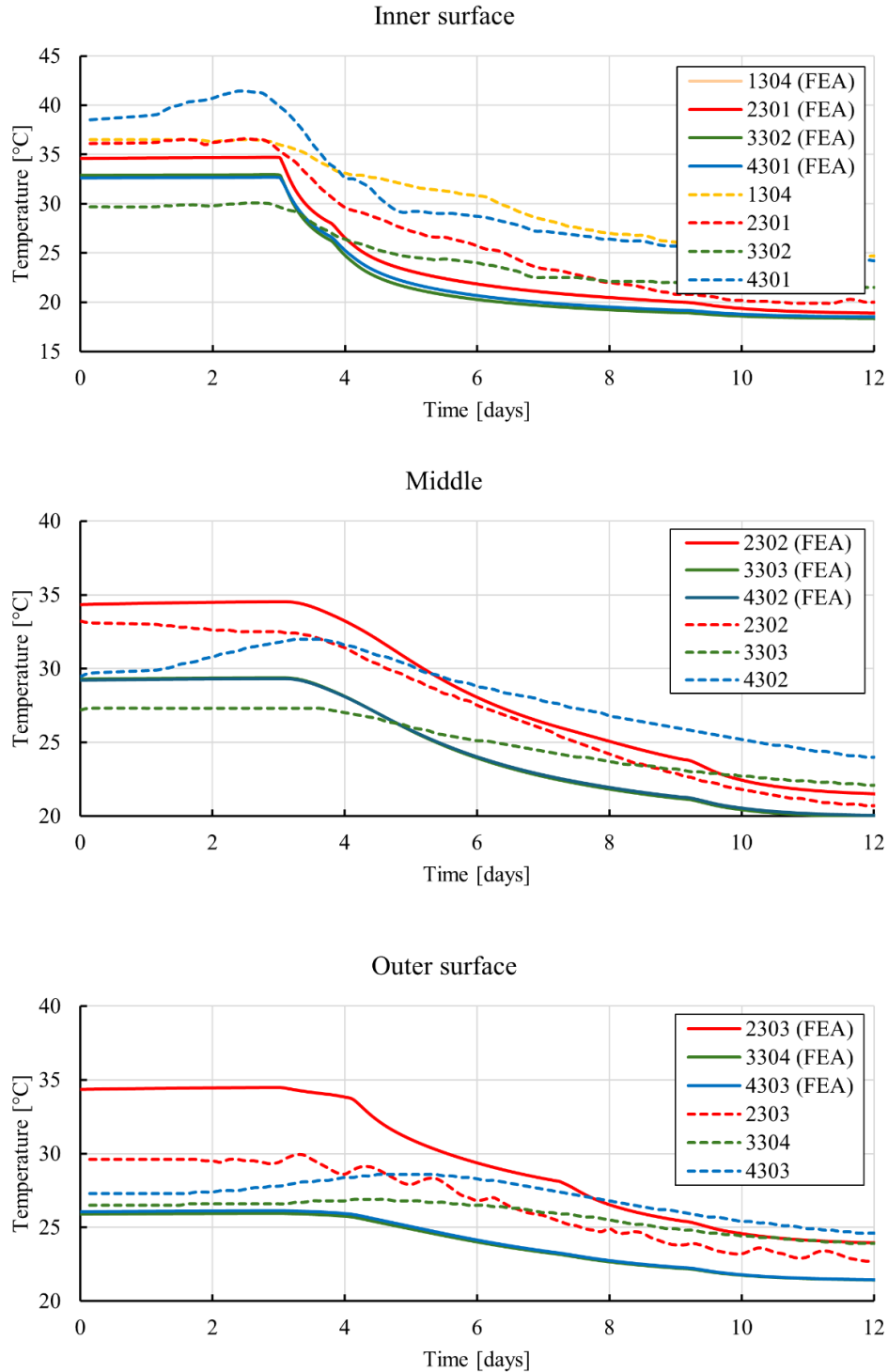


Figure 8. Comparison of the calculated heat transfer values with the measured values monitored on inner, outer surface and in the middle of the wall thickness.

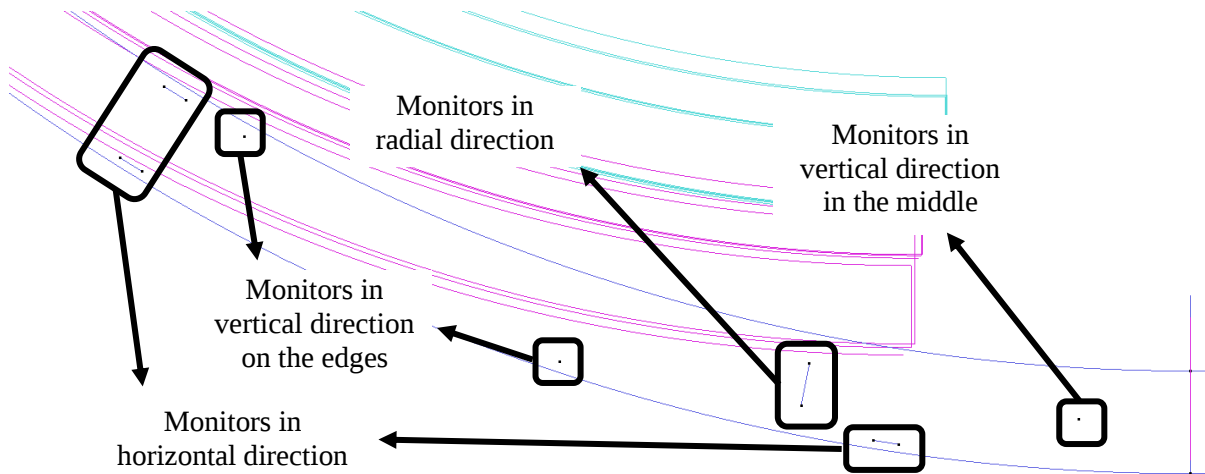


Figure 9. Illustration of the monitors in in model for tracking strains in the calculation.

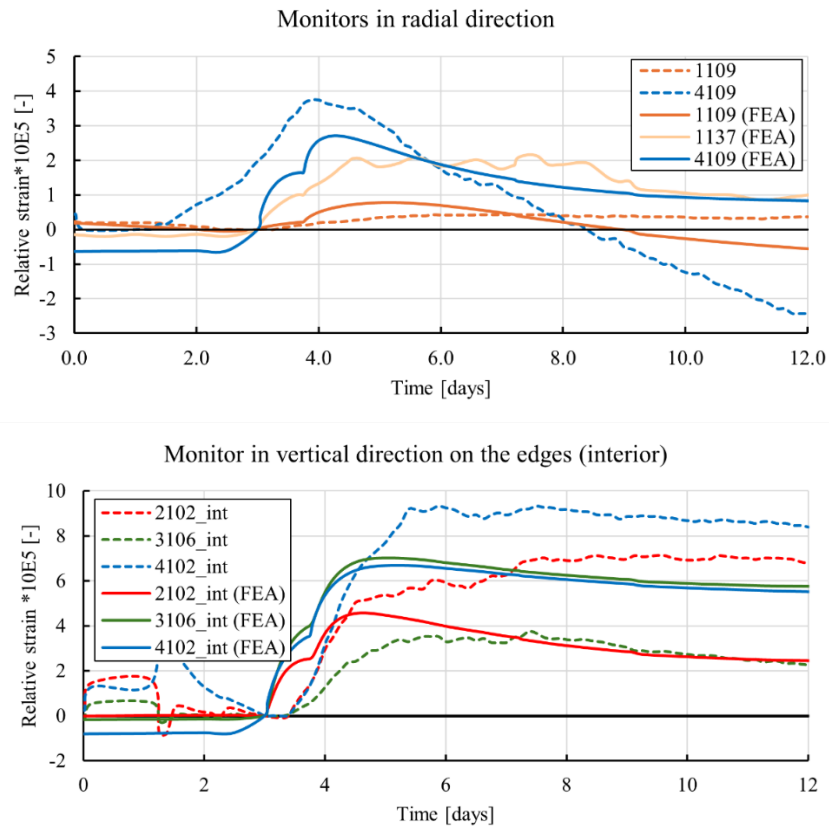


Figure 10. Comparison of the selected strain sensor in radial and vertical direction.

The calculation revealed that the highest tensile stresses occur at the location of the prestressing cables, as shown in Figure 11. Due to the cooling effect, a reduction in these values is observed. The development of cracks as the structure cools is shown in Figure 12. It is important to note that these are microcracks, with a magnitude on the order of tenths of a millimeter.

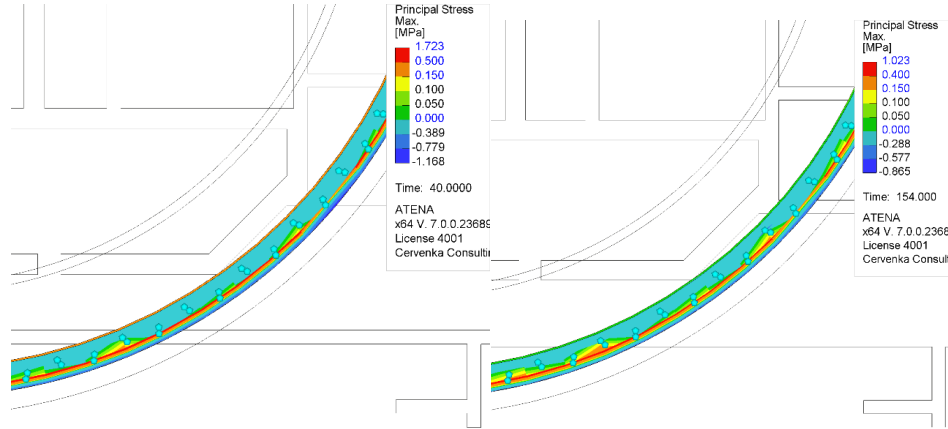


Figure 11. Containment wall tensile strains at the shutdown start (left images) and after 7 days (right images). Light dots indicate the locations of prestressing cables in the model.

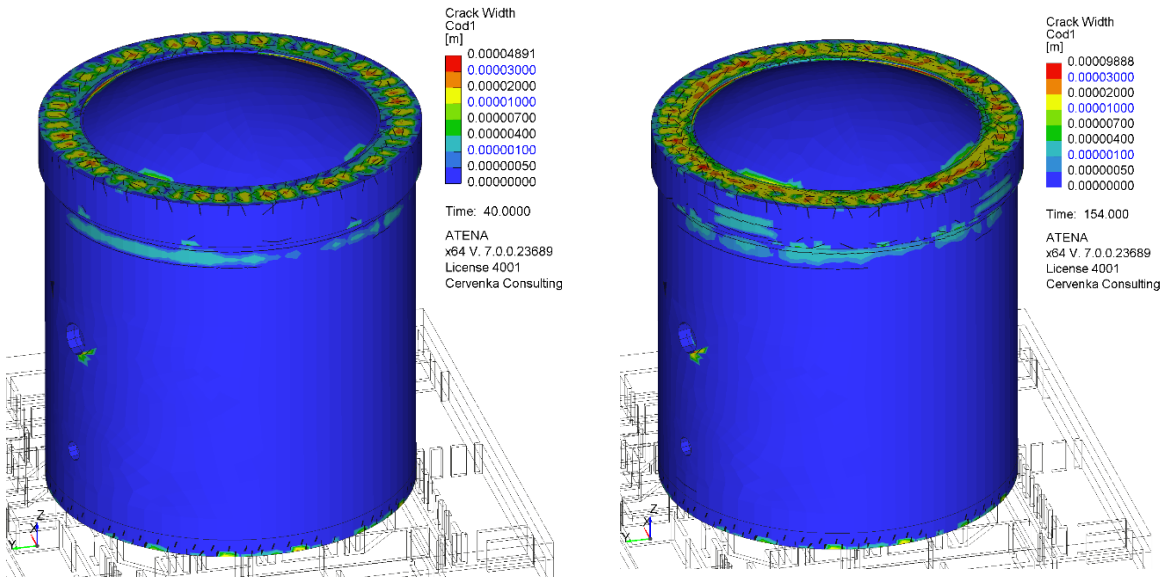


Figure 12. Crack widths in the containment, (left) cracks at the shutdown start, (right) after 7 days.

CONCLUSION

This paper discusses the application of nonlinear modelling and simulation in the concept of digital twins in the management of structural aging. The numerical analysis was performed using ATENA software, with heat transport analyzed by a transient thermal analysis and the resulting deformation response evaluated by a nonlinear static analysis. The primary focus was on calibrating the input parameters mainly for the thermal analysis based on temperature and strain sensor data.

The first part of the paper discusses the used material model and the evaluation process for the model uncertainty. An interesting insight into the model accuracy and validity is provided by an overview of 22 results from blind prediction competitions, where consistent predictions were obtained for the presented numerical model in typical failure modes involving shear, bending failure modes in reinforced as well as prestressed concrete members.

The paper presents initial results from the first cycle of the calibration and optimization process involving mainly the identification of thermal boundary conditions of the containment wall during the normal operation and during and after the operational shutdown. The calculated thermal strains were

applied as non-mechanical loading. The results show qualitatively good agreement with the development of temperatures as well as strains in the structure during the shutdown while the error in the actual values of calculated temperatures is approximately 30% and even higher for the obtained strain sensors. In the second stage of the optimization process this will be improved by the refined optimization of the thermal boundary conditions including also thermal parameters such as conductivity and specific heat. The stress analysis will be enhanced by considering long term behavior: creep, shrinkage and several cycles of the containment operation and shutdown.

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REFERENCES

- Bažant, Z.P. (1976). “Instability, Ductility and Size Effect in Strain Softening Concrete,” *J. Engrg. Mech.*, ASCE, Vol. 102, No. 2, pp. 331-344.
- Bažant, Z.P. and Oh, B.H., (1983). “Crack band theory for fracture of concrete,” *Materials and Structures, RILEM* 16 (3), 155–177.
- Červenka, V., Gerstle, K., (1971). “Inelastic analysis of reinforced concrete panels. Part I: Theory,” *Publication I.A.B.S.E.* 31 (11), 32-45.
- Červenka, J., Červenka, V., Eligehausen, R. (1998). “Fracture-Plastic Material Model for Concrete, Application to Analysis of Powder Actuated Anchors,” *Proc. FRAMCOS 3*, pp 1107-1116.
- Červenka, J., Janda, Z., Pryl, D. (2011), Numerical Simulation of Prestressed Concrete Containments, *Proc. SMIRT 21*, New Delhi
- Červenka J. and Papanikolaou V. (2008). “Three Dimensional Combined Fracture-Plastic Material Model for Concrete,” *Int Journal of Plasticity*. 24:2192-220.
- Červenka V, Červenka J, Kadlec L. (2018). Model uncertainties in numerical simulations of reinforced concrete structures. *Struct. Concrete*; 19(6): 2004.
- Červenka, J., Červenka, V., Laserna, S. (2018). “On crack band model in finite element analysis of concrete fracture in engineering practice,” *Eng. Fract. Mechanics*, Vol. 197, pp 27-47.
- Červenka V, Červenka J, Kadlec L. (2018). Model uncertainties in numerical simulations of reinforced concrete structures. *Struct. Concrete*; 19(6): 2004.
- Červenka, V., Červenka, J. and Jendele, L. (2025). “ATENA Program Documentation, Part 1: Theory,” Cervenka Consulting s.r.o. www.cervenka.cz.
- de Borst, R. (1986). Non-linear analysis of frictional materials. PhD Thesis, Delft University of Technology, The Netherlands.
- Menetrey, P. and Willam, K.J. (1995). “Triaxial Failure Criterion for Concrete and its Generalization,” *ACI Structural Journal*. 92:311-8.
- Ngo, D., Scordelis, A.C. (1967). “Finite element analysis of reinforced concrete beams,” *J. Amer. Concr. Inst.* 64, pp. 152-163.
- Proske, D., Kurmann, D., Cervenka, J. (2013), Seismische Tragfähigkeit eines Stahlbetongebäudes, *Beton- und Stahlbetonbau*, 108, Heft 8, DOI: 10.1002/best.201200080
- Rashid, Y.R. (1968). “Analysis of prestressed concrete pressure vessels,” *Nuclear Engineering and Design* 7 (4), 334-344.
- Rots, J.G., Blaauwendraad, J. (1989). Crack models for concrete : Discrete or smeared, Fixed, multi-directional or rotating? *Heron* 34 (1).
- Kučerová A., Sýkora J., Havlásek P., Jarušková D., Jirásek M. (2023). “Efficient probabilistic multi-fidelity calibration of a damage-plastic model for confined concrete,” *Computer Methods in Applied Mechanics and Engineering*, Vol. 412, ISSN 0045-7825