COMPUTER SIMULATION OF FAILURE OF CONCRETE STRUCTURES FOR PRACTICE

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Keywords: non-linear analysis, fracture mechanics, plasticity, failure, pre-stressed concrete

1 INTRODUCTION

Non-linear analysis of concrete structures became a novel design tool. It employs the power of computer simulation to support and enforce the creativity of structural engineers. Although it is frequently used in research and development its potential for engineering practice has not been yet fully discovered. In the new design standards for concrete structures, such as Eurocode 2 (1992), the non-linear behaviour of concrete is described by a uni-axial stress-strain diagram. However, this is only a fraction of the available knowledge of material models and theories discovered and validated by research. The finite-element-based failure analysis can take advantage of rational and objective theories such as fracture mechanics, plasticity and damage mechanics. It makes possible a "virtual testing" of building structures under designed loading and environmental conditions. The implementation of these advanced methods and techniques introduces new possibilities for the engineer, not only in research and development but also in practical design. Computer simulation is a modern tool for optimisation of structures, which can contribute to better economy in design.

The author has been involved in the research of this subject over 30 years and has published many original contributions, among which his dissertation reported in ref. [1] and winning contribution to the Toronto Competition [2] were widely recognised. At present he is heading the development of the finite element software ATENA.

The goal of this paper is to present recent experiences with the computer simulation of concrete structures and its potential for practical engineering work. The principles of the theoretical background will be briefly mentioned and illustrated by validation examples. The third part of the paper is focused on application areas.

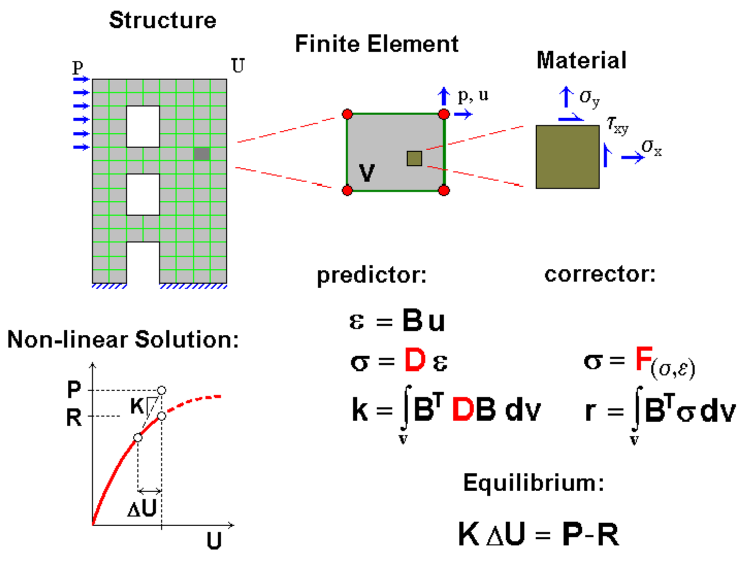


Fig. 1 Scheme of the non-linear finite element algorithm.

2 NON-LINEAR FINITE ELEMENT ANALYSIS

2.1 Non-linear analysis

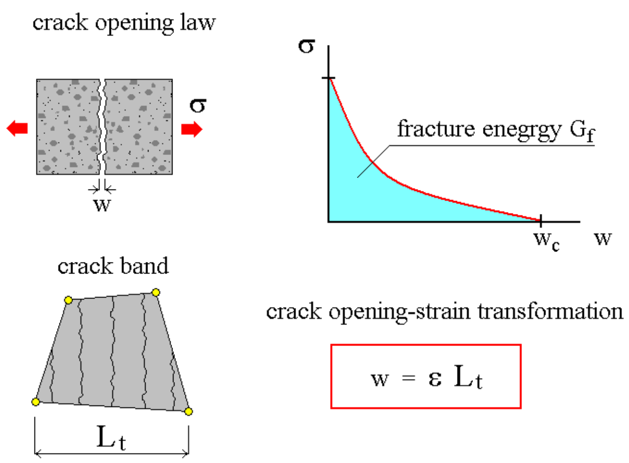
This paper extends the basic ideas of this approach presented during the *fib* Symposium 1999 [3]. Non-linear analysis can be characterized by several levels of numerical modelling as shown on the solution algorithm in Fig.1. The finite element technique is used for structural modelling in which the basic matrix equilibrium equation works with the nodal force vector **P**, nodal displacements vector **U** and the stiffness matrix **K**. Material properties are described by the constitutive relations between stresses and strains $\sigma = \mathbf{F}_{(\sigma,\varepsilon)}$. These material relations for concrete are highly non-linear. The straindisplacement relation described by the matrix **B** is also non-linear (and represents a second order effect).

The solution of the non-linear equations is performed by non-linear solution techniques based on the iterative predictor-corrector scheme, whereas the load history is applied in load steps (increments) and iteration is performed in each step until the equilibrium is satisfied. Thus, at the end of iteration the applied load **P** and the structural resistance **R** should be equal within an accepted tolerance. The equilibrium equation is linear within one iteration, but it is non-linear within the global solution.

Let us look at the roles of different building stones of this model. On one side, the constitutive relation $\sigma = \mathbf{F}_{(\sigma,\varepsilon)}$ in a material point plays the most crucial role and decides how the model represents reality. On the other side, the requirements on the stiffness matrix **K** are quite relaxed. It is only a numerical tool for the "predictor" and should guarantee a convergence of the iterative solution. For example it can be the elastic isotropic material law. Last but not least, the size of finite element mesh is an important factor and should be under control. As shown, the numerical solution includes many approximations and thus it is reasonable to maintain a balanced level of approximation in all levels.

2.2 Constitutive models of concrete structures

Concrete is a complex material with strongly non-linear response even under service load conditions. Therefore, special constitutive models for the finite element analysis of concrete structures have been developed. We shall briefly described some of these models implemented in ATENA [4],[5].



Tensile behaviour of concrete is modelled by non-linear fracture mechanics combined with the crack band method. In this method the smeared crack concept is accepted. Its parameters are: tensile strength, shape of the stress-crack opening curve and fracture energy, see Fig.2, based on ref. [6],[7],[8]. In this formulation the crack strain is related to the element size. Consequently, the softening law in terms of strains for the smeared model is calculated for each element individually, while the crack-opening law is preserved.

Fig. 2 Crack opening law according to Hordijk [8] (above). Crack band model (below).

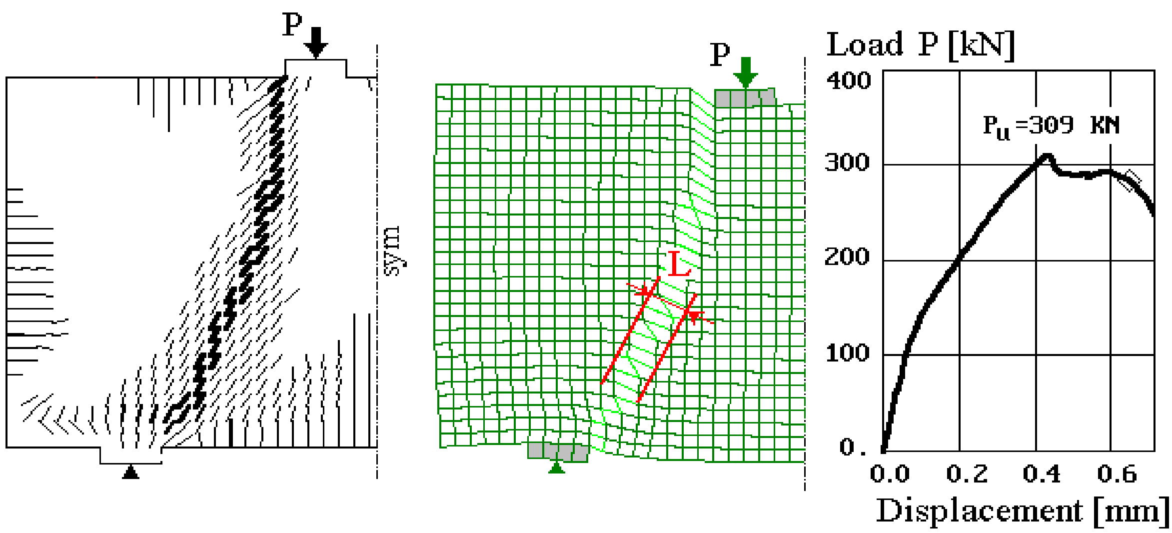


Fig. 3 Example of a crack band in the shear wall.

A real discrete crack is simulated by a band of localized strains as illustrated in Fig.3. Due to the energy formulation this model is objective and its dependency on the finite element mesh size is substantially reduced [9]. This was confirmed by numerous studies, for example by those about shear failure published in [10].

Two crack models are recognized. In the *fixed crack model* the crack direction is determined and fixed at the time of crack initiation. In the *rotating crack model* the crack direction is identical with a principal strain direction and rotate if the strain direction changes. The main difference in these crack models is the absence of shear stresses on the crack plane in the rotating crack model due coincidence of principal strain directions with the crack orientation, which makes the rotating crack model more simple. In the fixed crack model the shear resistance of the cracks is modelled by means of the variable shear retention factor, which reflects the aggregate interlock effect of cracked concrete.

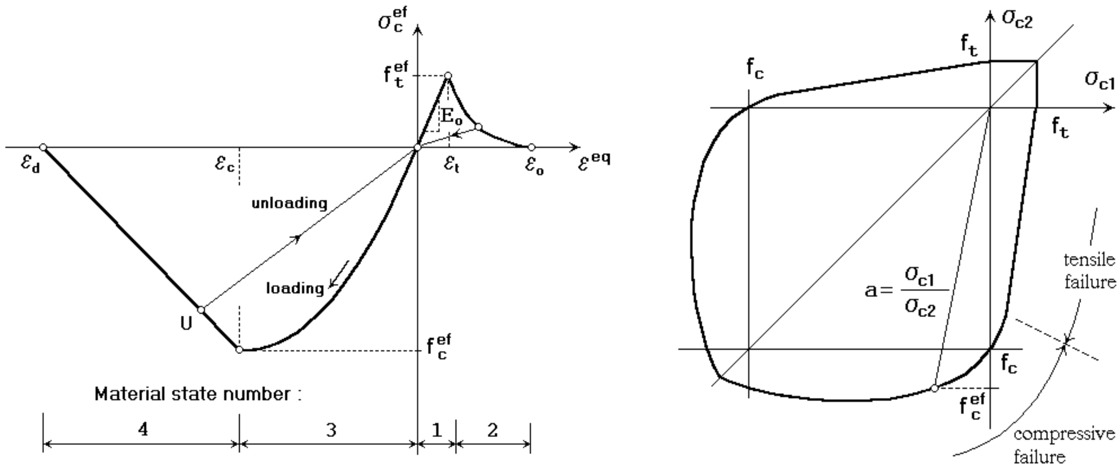


Fig. 4 Equivalent uniaxial law. (left) Bi-axial failure function by Kupfer [11]. (right)

Concrete in plane stress condition can be well described by a damage model such as the one used in the ATENA, see Fig.3. It is based on the “equivalent uniaxial law”, which covers the complete range of the plane stress behaviour in tension and compression. The effect of biaxial stress state on the concrete strength is captured by the failure function due to Kupfer et al [11]. For the tensile response (cracking) the crack band method described above is applied. Similar method is applied for the compressive softening. Thus complete softening behaviour is based on an objective and mesh independent approach.

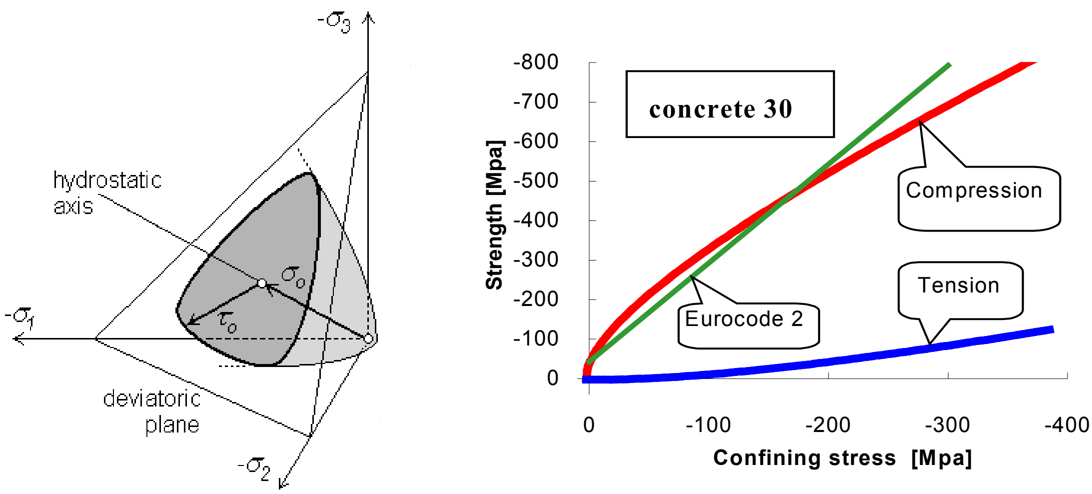


Fig. 5 Concrete failure in 3D stress state (left). Strength increase under confinement effect (right).

Concrete under confinement in three-dimensional stress state can be described by the theory of plasticity with a non-associated flow rule according to Menetrey-Willam, [12], [13]. The failure function is illustrated in Fig.5 (left) in the space of principal stress. The strength is increasing with the hydrostatic compressive stress σ_0. The failure occurs when the deviatoric shear stress τ_0 reaches the failure function. An example of strength increase in the case of Menetrey-Willam failure function is shown in Fig.5 (right) and is compared with the similar function recomended by the Model Code 90. Strength is the maximum stress in σ_3 under the assumption of confining pressure σ_2=σ_1. This graph represents a section through the surface shown in Fig.5 (left).

The plasticity theory describes also the plastic flow and volume change due to a distorsion and can model the volume increase of concrete under plastic deformations. The predictor-corrector scheme, Fig.1, is used for this purpose. The plastic volume change can be defined in the rendulic section, Fig.6, where ξ is the hydrostatic axis and ρ the deviatoric axis. Parameter β defines the return direction form the predicted stress point σ_{ij}^{t} back to the failure surface. This direction is used in the plastic flow rule to define the direction of plastic flow (in the principal stress space) and describes the volume change.

It should be noted that the general three-dimensional behaviour of concrete can be also described by the microplane models, which are based on a micro-structural damage idea. Microplane model due to Bazant [14] is implemented in ATENA and can be used for solution of practical cases.

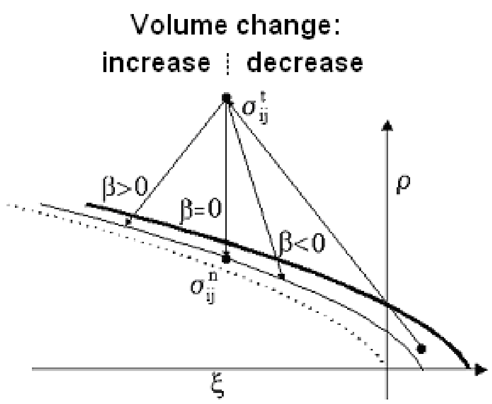


Fig. 6 Return mapping algorithmus.

Of course many other material models are needed for completeness of a successful simulation (reinforcement, steel, bond slip of reinforcing bars, interface, soil, etc.). Out of these models we shall mention only bond since it represents a specific feature of reinforced concrete mechanics.

Bond serves to transfer the stresses from reinforcement to concrete and assures the integrity of reinforced concrete structures. Bond mechanics is very complex and include all behaviour modes of concrete: friction, tensile and splitting cracks, compressive crushing. Theoretically it should be possible to solve the bond with available solid models. However, such approach leads to very large tasks and is not practically useful.

High attention was paid to develop more simple bond models, see for example the FIB report [15]. These models are usually cast in form of bond stress-slip diagrams and include influencing parameters (concrete properties, confinement, surface properties). They can be directly used in the finite element models. Examples are shown in Fig.7. It is realized, that some features, which are covered by the simplified bond laws are already covered by the concrete models. This is especially true for concrete splitting, which is the main source of the softening behaviour of bond slip laws. Such aspects must be carefully considered in particular cases. For example, in case of analysis, which sufficiently covers the splitting of concrete by a detail analysis of cracks in concrete it may be adequate to choose the plastic bond model.

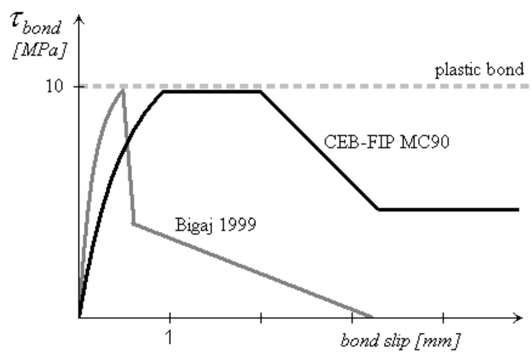


Fig. 7 Examples of bond slip law.

2.3 Elements of reinforced concrete model

Finite element modelling of reinforced concrete structures requires special tools for modelling of all types of reinforcement. Most of these models are illustrated in Fig.8 and 9 for two- and three-dimensional solids. In addition there is a family of similar axisymmetric elements, which is not shown here.

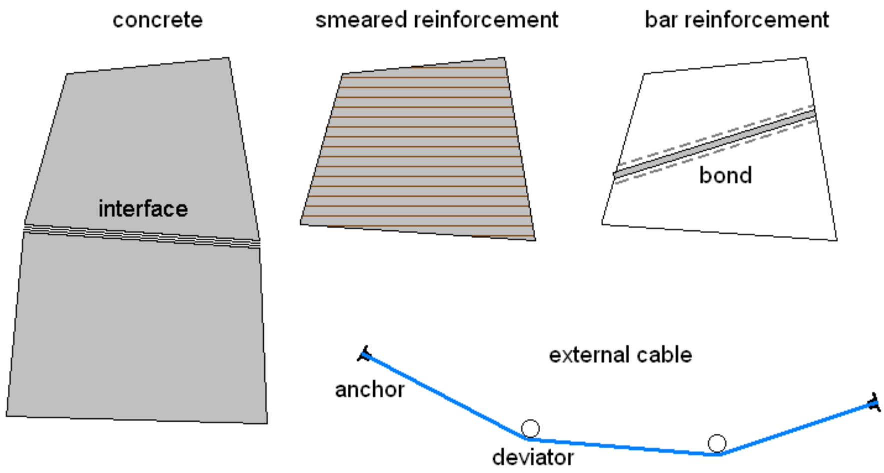


Fig. 8 Two-dimensional elements of reinforced concrete.

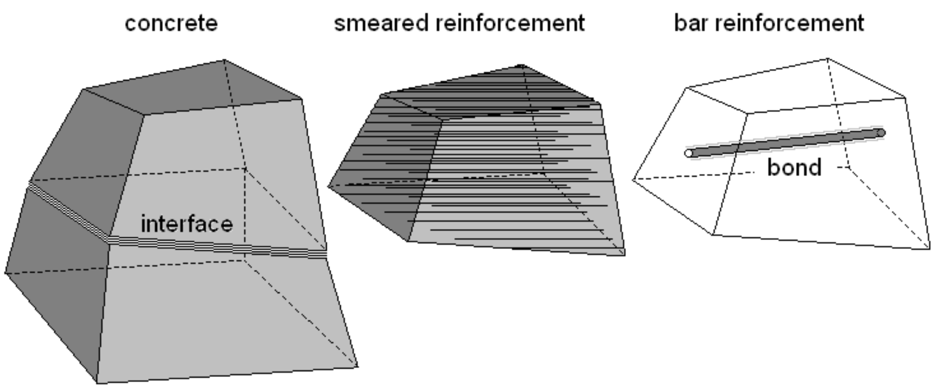


Fig. 9 Three-dimensional elements of reinforced concrete.

Concrete is modeled by solid elements. As examples quadrilateral and brick elements for 2D and 3D solids are shown. Low order elements are recommended for nonlinear analysis, since they are well validated. Concrete to concrete, or concrete to other material interfaces can model the frictional type of interaction between structural elements. The mesh type of reinforcement can be represented as smeared reinforcement. In this element the individual bars are not considered, while reinforcement is considered as a component of the composite material.

Individual bars can be modelled by truss elements embedded in concrete elements with axial stiffness only. In this technique the mesh is generated first for concrete. Then the bar elements are embedded in this mesh. The bar element can be considered as any other element but its nodes are made kinematically dependent on concrete nodes. Thus the reinforcing is not affecting the mesh generation. Cables, which are connected with the concrete structure only in limited number of points can be modelled as external cables. The described family of finite elements make possible a to cover most practical cases of reinforced concrete structures.

3 VALIDATION

It is important that before they are applied in practice, programs are first tested and validated by all features mentioned in concrete comparing them with well-defined and documented experiments. Such tests can be quite demanding when performed without a prior knowledge of the experimental results at the time of analysis, which is often a requirement. For illustration we shall present here two cases of validation dealing with crack propagation and crack width, which are fundamental for design of serviceability states.

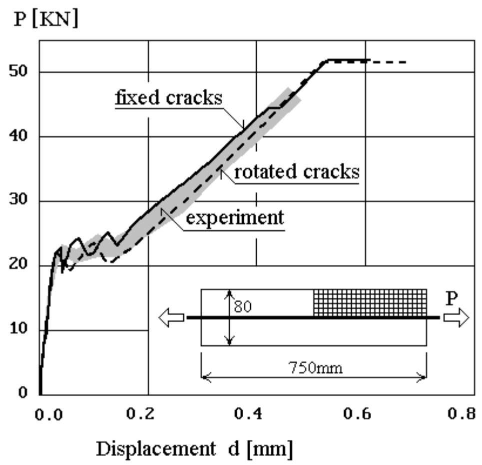


Fig. 10 Uniaxial tension test. Simulation of experiments by Hartl [16].

In the first example, Fig.10, a concrete member is reinforced by a steel bar and subjected to an axial tension. This test was done within the research of tension stiffening effect performed by Hartl [16]. The aim of the research was to establish the effect of concrete contribution to the stiffness of the reinforcing bar after cracking and here it is used to validate the ATENA crack models. This example shows that the agreement of the model response with the experiment is very good. The contribution of concrete to the stiffness of the steel bar is generated by the regions of uncracked concrete. These regions are specific for each structure and are precisely created in each case. Therefore, the tension stiffening effect is simulated very realistically in this approach and is not dependent on geometry and reinforcement.

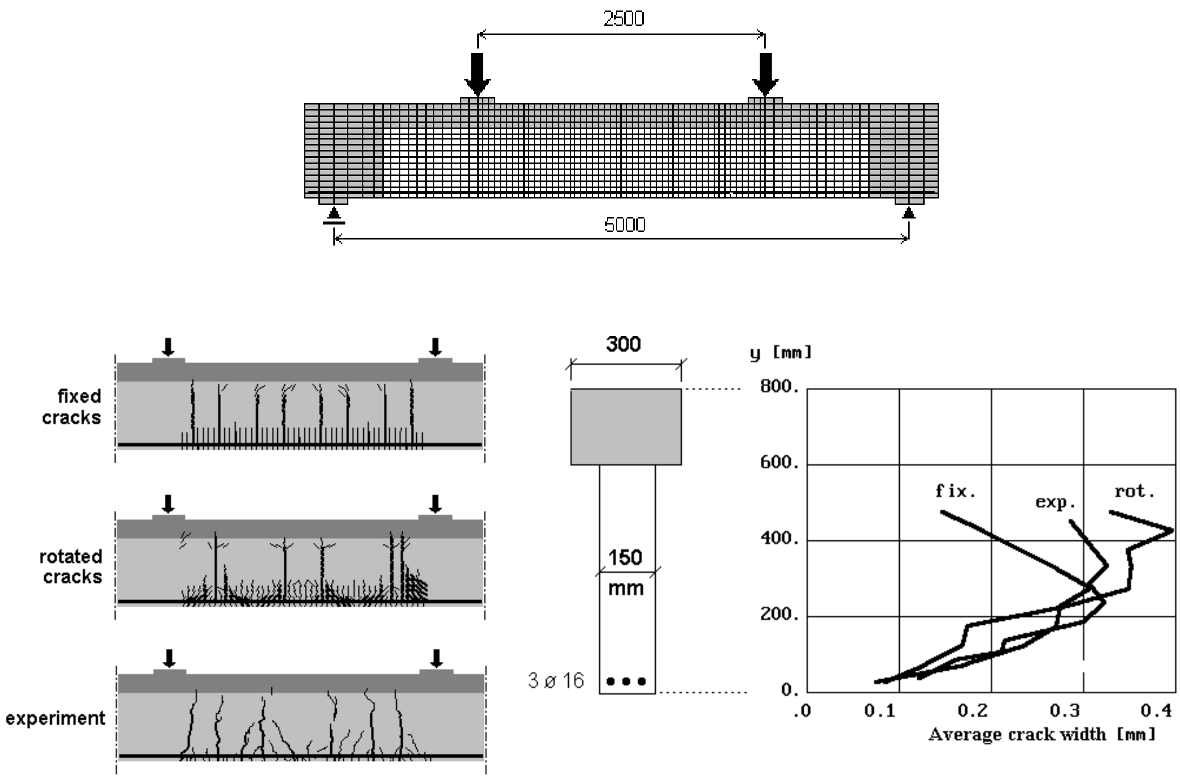


Fig. 11 Simulation of cracks in the web of beam tested by Braam [17]. Top: finite element model. Left: calculated and experimental crack patterns. Right: Average crack widths.

The second example, Fig. 11, shows the simulation of crack patterns and crack widths in the web of a reinforced concrete beam tested by Braam [17], [18]. As expected, the crack pattern is strongly affected by reinforcement. The experimental cracks width is smaller near reinforcement and grows with a distance from reinforcement. The inverse is true for the number of cracks. The simulation follows closely this behaviour, whereas the rotating cracks give greater crack spacing far from reinforcement.

Similar validations were performed also for other failure modes, such as shear failure of beams [10] and shear walls [5].

4 APPLICATIONS

4.1 Reinforcement near openings

Engineer often faces a problem, which is difficult to solve by ordinary design tools. A typical such problem is reinforcement detailing near openings shown in the next example of a large girder in a recently built shopping centre in Prague.

Fig. 12 Reinforcement detailing (left). Finished girder with opening (right).

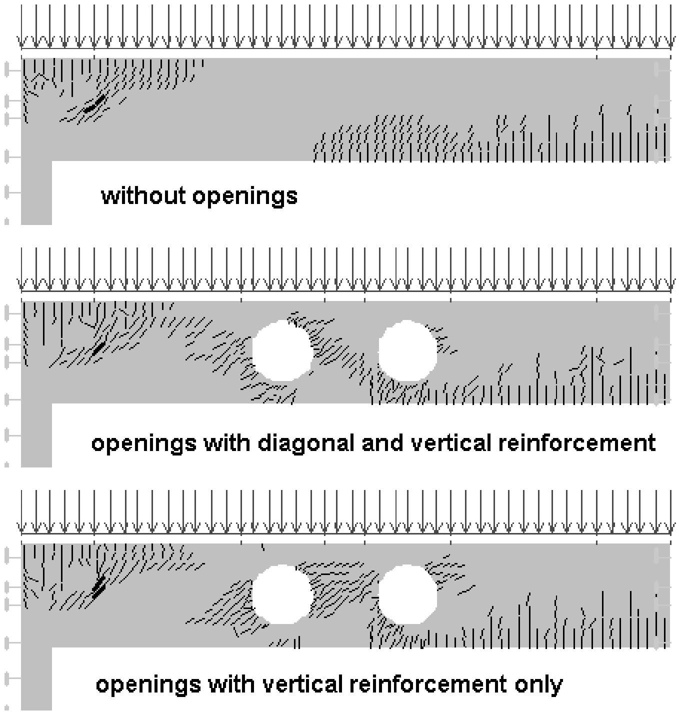


Fig. 13 Crack patterns in different design solutions.

The installation of a technical facility required rather large openings in the girders in locations exposed to shear forces. Fig. 12 shows the reinforcement detailing before concrete casting and the finished girder structure with installed air-conditioning pipes in openings.

In order to verify design the girder was analysed by the program ATENA. The analysis included a study of various opening shapes and reinforcement detailing. Examples of calculated crack patterns are shown in Fig. 13 where the crack widths under service loads were evaluated. The structural performance was checked on the load-displacement diagram, Fig. 14. Here the deflection under the service load and the ultimate load capacity can be compared for various design solutions. The study has proven, that the girder with circular openings provided with diagonal reinforcement can resist higher limit loads than a comparable girder without openings. The crack width under the service load in this case was shown to be below the limit value of 0.3 mm.

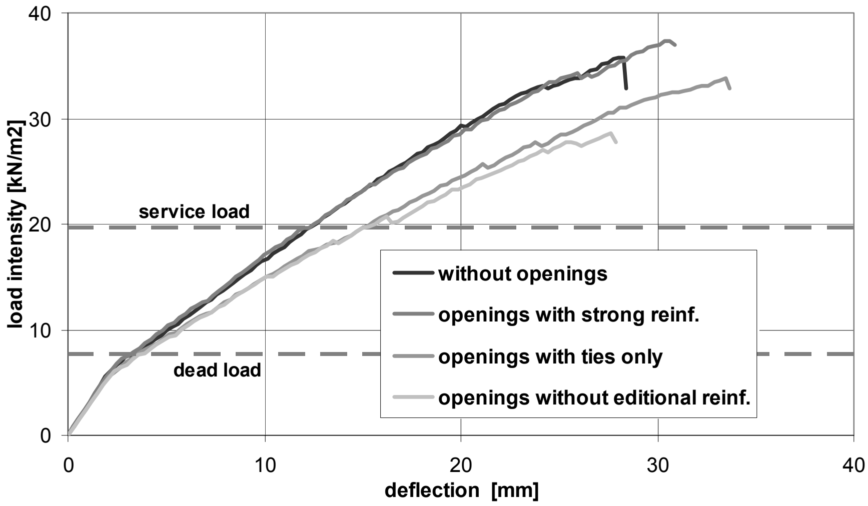


Fig. 14 Load-displacement diagrams for different solutions.

4.2 Slab punching

The next example, Fig. 15, shows a case of plate punching, where the three-dimensional stress state is an important factor for determining the ultimate load.

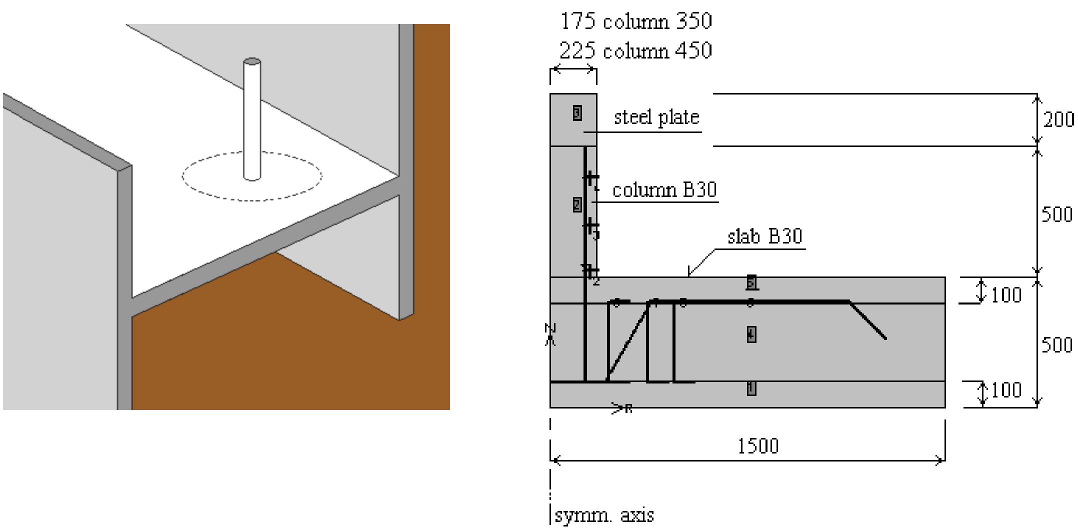


Fig. 15 Column-slab joint (left). Axially-symmetrical model (right).

The purpose of the analysis was to check if the reinforcement detailing is adequate to prevent a punching failure. For simplicity the model is assumed in a state of rotationally symmetrical stress state. The plate is provided by horizontal, vertical and diagonal reinforcement near the column head. The horizontal reinforcement is in both radial and circumferential directions. The plate is vertically supported on its perimeter and loaded on the top of column by vertical force. For the concrete material the 3D-fracture-plastic model is chosen, which is based on a combination of two failure theories, namely: fracture mechanics (fracture energy based crack band method) for tension and Menetrey-Willam failure surface for compression.

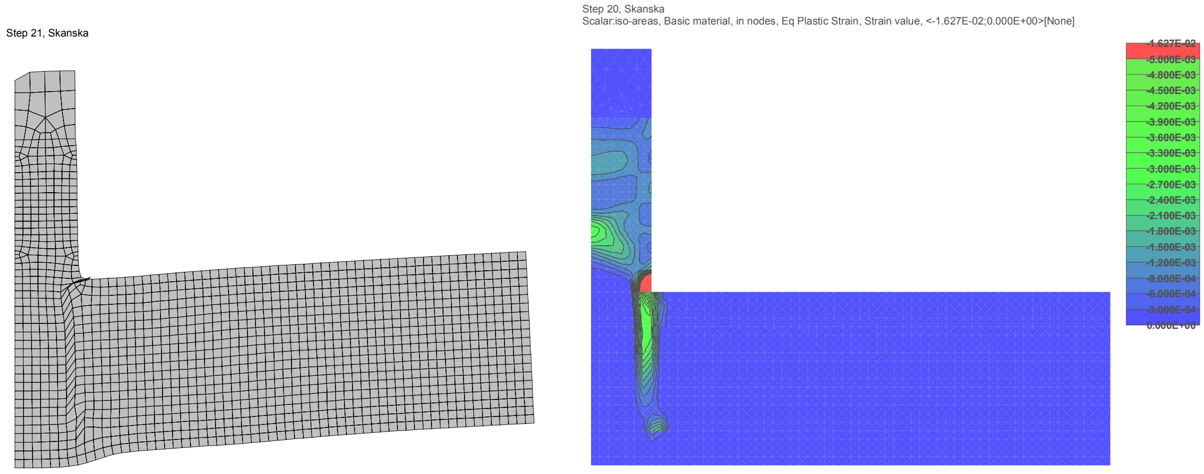


Fig. 16 Punching failure. Deformed mesh, factor 50 (left). Iso-areas of plastic strains (right).

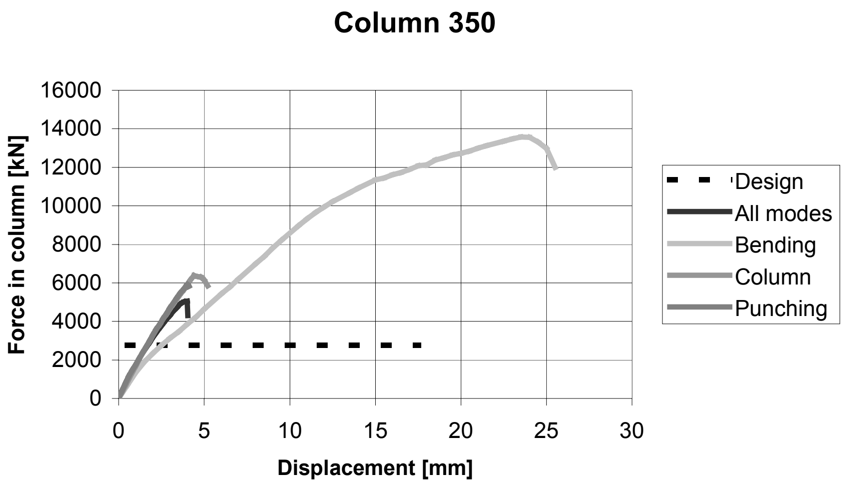


Fig. 17 Load-displacement diagrams of different failure modes.

The simulation supported by variety of numerical and graphical results made it possible to determine the maximum load as well as the punching failure mechanism, as illustrated by Fig. 16. Based on these results a failure plane in the form of a cylinder around the column head could be detected. The failure is due to the plastic sliding on this plane and provides the minimum failure load.

Theoretically there are several failure mechanisms, which come into consideration: diagonal tension (cone-failure surface) failure, punching (cylinder failure surface), column compressive failure, plate bending failure. In the simulation all these modes can be easily isolated and evaluated, as shown in Fig. 17. For example a column failure is isolated, when the plate is considered with artificially stronger or linear-elastic material. In an unconstrained analysis of this particular case, where all models are allowed, the punching mode gives the lowest resisting load.

The study proved that the design of reinforcement in this particular case was inadequate and did not prevent a brittle mode of punching failure. It was also shown that by a small change in location of the same amount of reinforcement the punching mode of failure could be prevented or its load capacity could be substantially increased.

Frame corner

Frame corner is a typical design problem, where reinforcement detailing plays important role. The following example is taken from the project of the newly built Brixen University designed by the Consulting Office Bergmeister, Brixen. The non-linear analysis was performed by ATENA in order to optimise the reinforcing in the frame corner under the constrain imposed by the crack width limit. The girder is pre-stressed and the cable anchors are significantly influencing the stress state in the frame corner. The model for analysis is reduced to a section including the symmetrical half of the girder with the floor slab and parts of columns, Fig. 18, Fig. 19. The model was development was based on the analysis of the whole 5-storey frame.

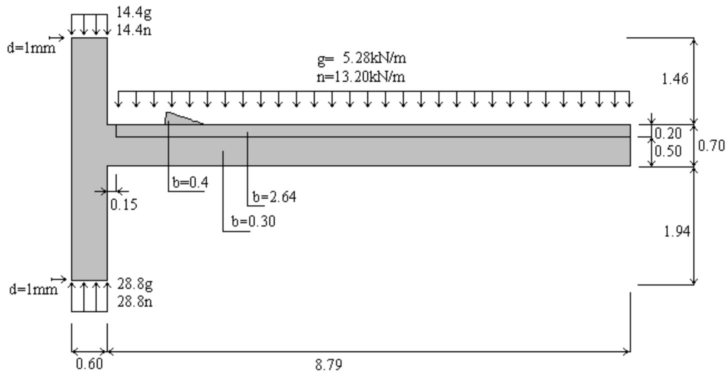


Fig. 18 Frame corner with dimensions and loading. (b – thickness)

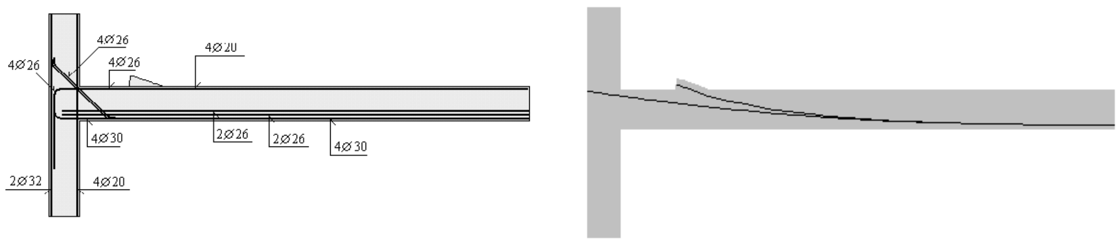


Fig. 19 Reinforcing by ordinary bars (left) and pre-stressing cables (right).

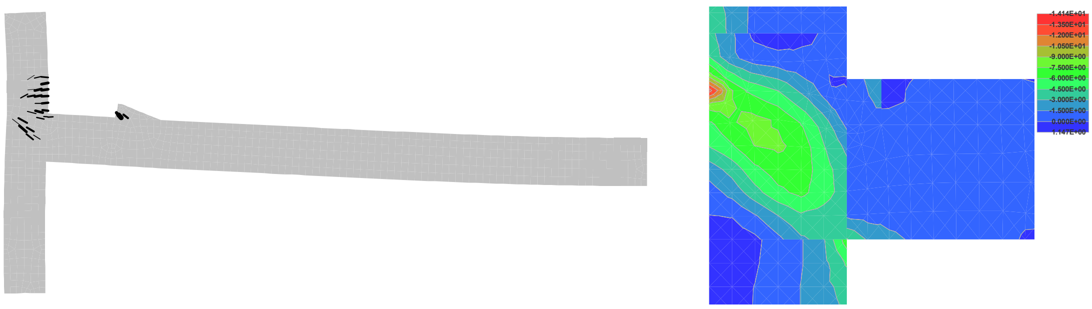


Fig. 20 2D analysis of service load.

Crack pattern and deformed shape (left) and compressive stress field in the frame corner (right).

Both, 2D and 3D solid analysis were performed. The study included all important loading and time effects (pre-stressing permanent load, live load, shrinkage and creep). Here we shall present only examples of results to illustrate this application. As first the results of 2D analysis in plane stress state are presented. The crack state and compressive stress field under service load are shown in Fig. 20.

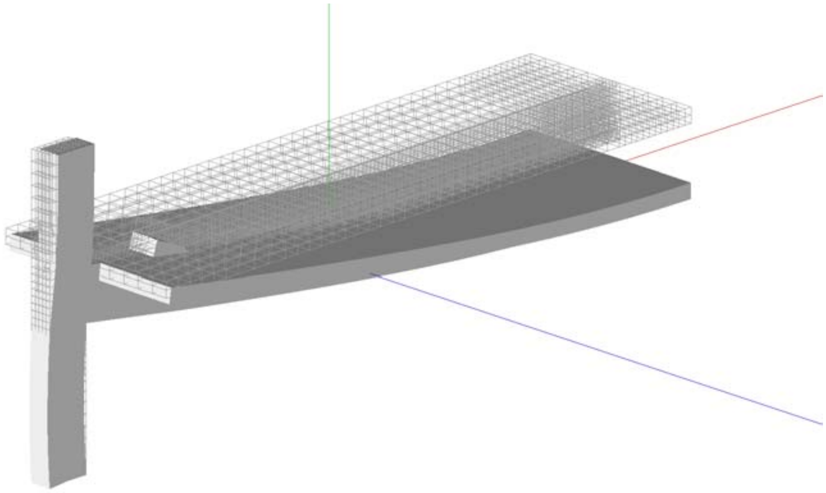


Fig. 21 3D analysis, deformed shape under service load.

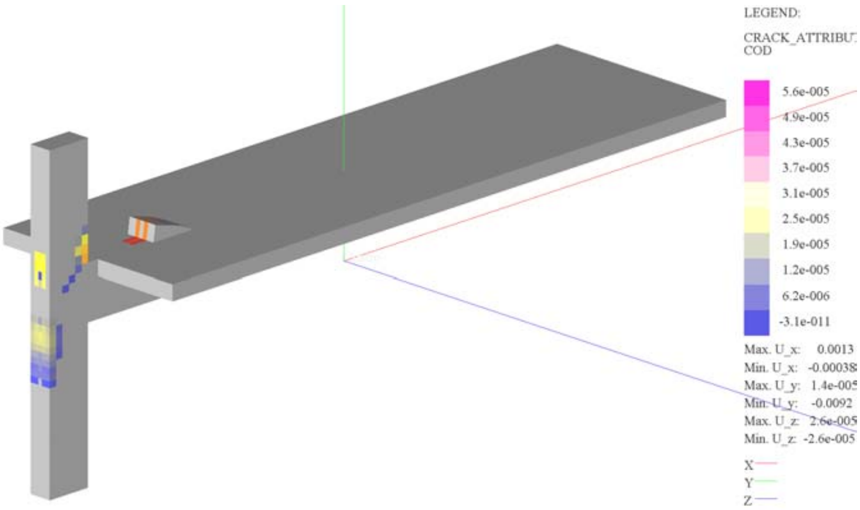


Fig. 22 3D analysis, crack widths under service load.

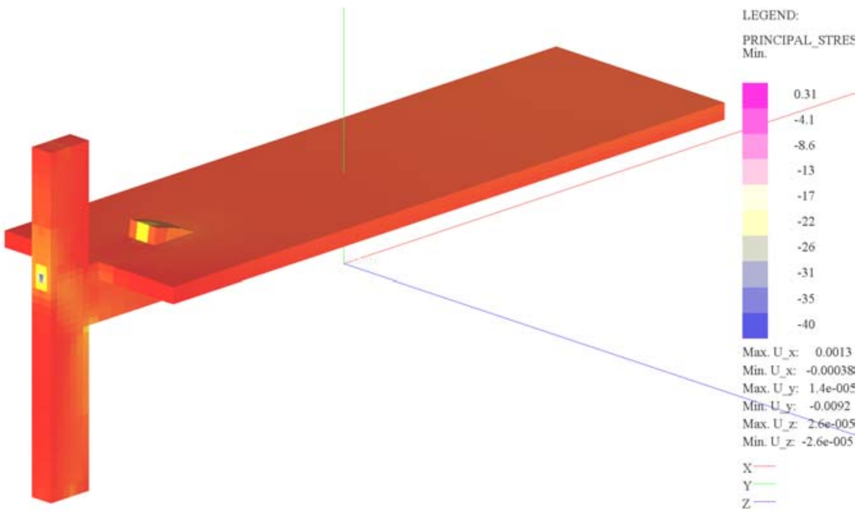


Fig. 23 3D analysis, compressive stress field.

Then the example of a 3D analysis is shown in the following figures. By comparing the pictures of compressive stress fields in 2D and 3D analysis, an effect of confinement can be demonstrated. In the plane stress analysis, Fig. 20, the compressive stresses are extended through the width of the beam-column joint and are not constrained in the third direction, while in the 3D analysis, Fig. 23, they are concentrated around the anchor and can be balanced by the horizontal reinforcement in the transverse direction. The study confirmed, that the 2D analysis is sufficient for the cases without significant three-dimensional stress states. However, especially in cases of anchoring regions of pre-stressing cables, where a confinement effect influences significantly the structural behaviour, a 3D analysis better reflects the reality.

Fig. 24 View of the Nusle Bridge in Prague.

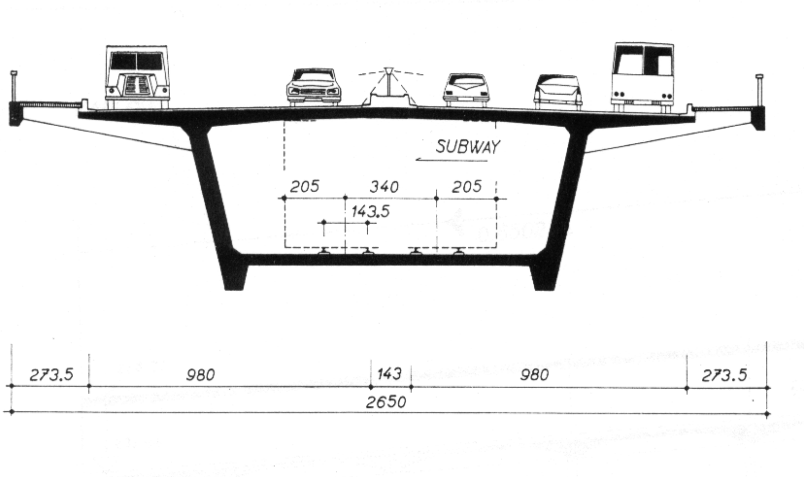


Fig. 25 Box girder cross sections. (Dimensions in cm.)

4.3 Box girder bridge

The largest bridge in the city of Prague was built in 1972 over the Nusle valley to serve a major six-lane highway on the top level and the two-way subway line inside the box, Fig. 24, Fig. 25. The bridge was recently investigated for safety evaluation. The simulation of loading response by ATENA was a part of this investigation. One of the main concerns was the effectiveness of the vertical prestressing in the box walls, which was expected to be substantially reduced due to corrosion. The study was performed as a part of the research conducted by J.Niewald under the guidance of Prof. V.Kristek at the Department of Civil Engineering of Czech Technical University in Prague, [19].

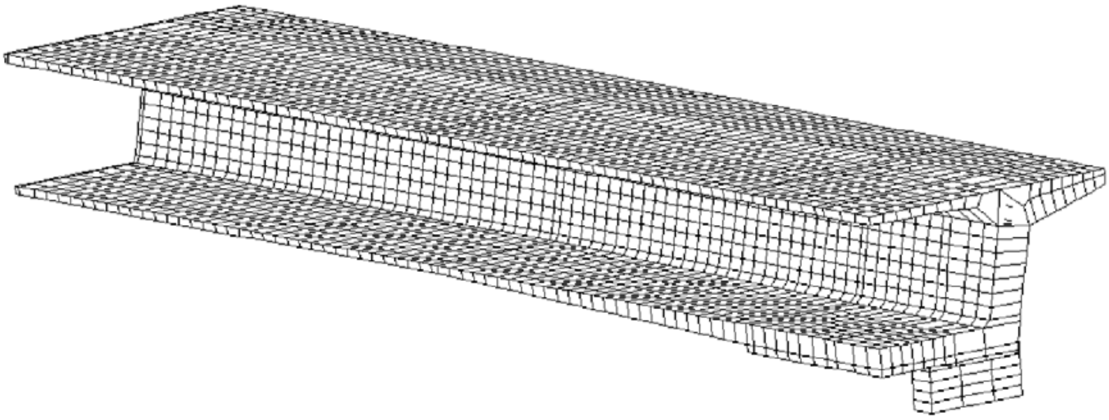


Fig. 26 Finite element model of the bridge.

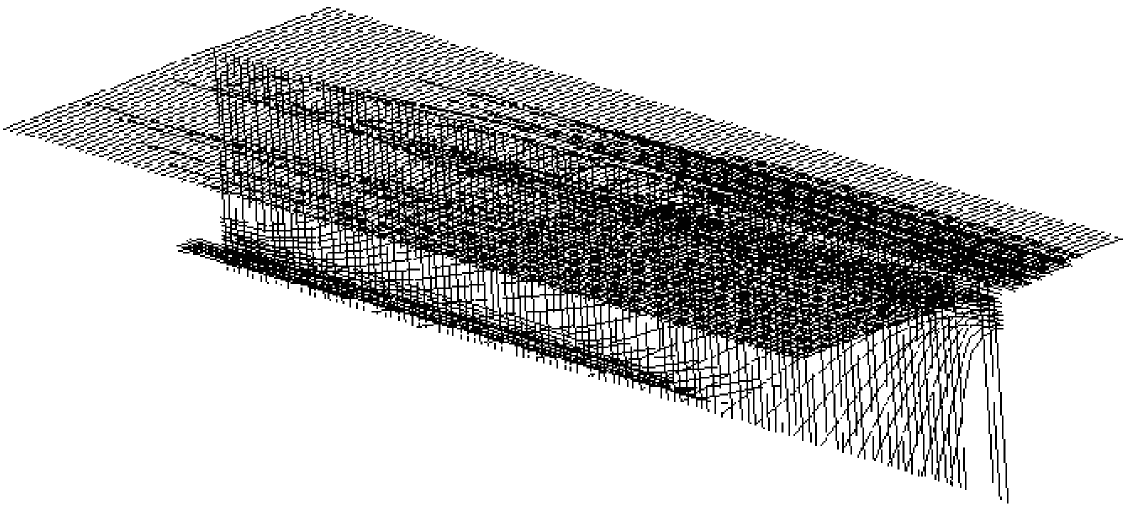


Fig. 27 Prestressing cable elements in the finite element model.

The structure of the bridge is a box girder with three main spans of 115 m and a total length of 485 m. The cross section is shown in Fig. 25. The main spans of the bridge were constructed by the cantilever segmental method. The bridge was pre-stressed by cables in the longitudinal vertical and transverse directions. The analysis model covers a half of the span with three symmetrical planes as boundaries, Fig. 26. The model of pre-stressing cables is shown in Fig. 27. In addition to the concrete and cable elements the ordinary reinforcement was included in form of smeared reinforcement layers.

The model was quite complex and reflected well the geometry, reinforcement and supports. The construction process and staged construction were not modelled in detail and were approximated in an average and simple manner, with the aim to estimate the current bridge properties. Since an exact estimate of the actual properties was difficult it was necessary to perform a parametric study to cover the range of expected parameters. This concerned mainly the vertical pre-stressing.

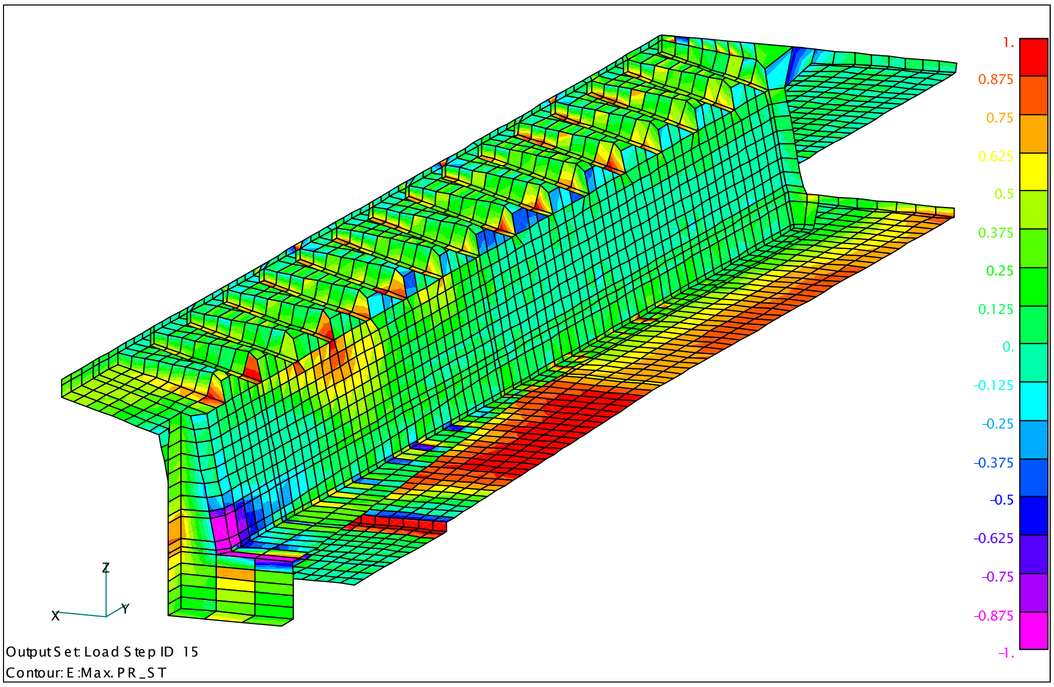


Fig. 28 Principal tensile stresses in concrete at service load.

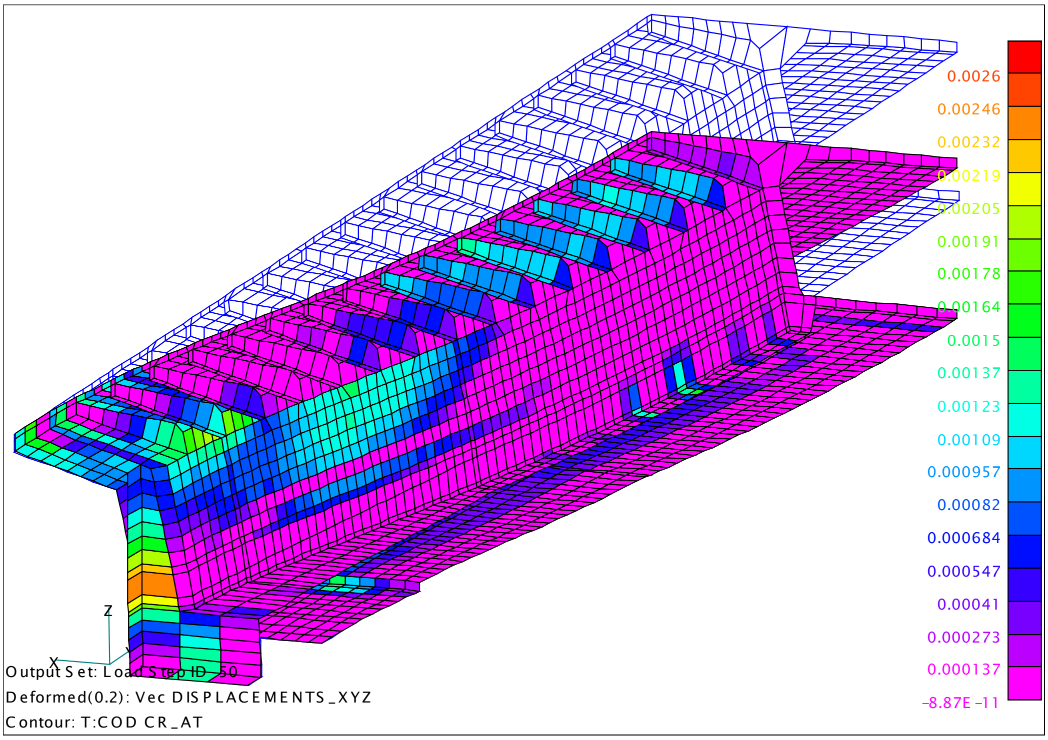


Fig. 29 Crack widths and deformed shape at the ultimate load.

In the analysis the load was applied in increments and reflected the actual load history. The states of particular interest were the service load and the ultimate load. Two examples of results are shown to illustrate the results of the study. shows the iso-areas of principal tensile stresses on the surface of the structure under the service load. At this stage all stresses are well under the tensile stress limit and no cracks are generated. The load was further increased up to the maximum load. In the ultimate stage the deformations increased dramatically and indicated the collapse mechanism. The deformed shape of the bridge with indication of crack opening near the collapse is shown in Fig. 29. Of course the crack width is not important at the maximum load but it is useful to check the failure mode. The maximum load was be used to evaluate the bridge safety.

The study has proven that the bridge performance under service conditions is satisfactory. The vertical pre-stressing influences the stress state in the walls of the box, but there is not a danger of cracking even if it is reduced near to zero. The safety factor of the bridge against the overloading by traffic load (live load) was found as $s_l = p_{l,ult}/p_{l,serv} = 7$ (where $p_{l,ult}$ and $p_{l,serv}$ are the ultimate and service levels of the live load, respectively. This is very high value, but it should be considered in the relation to the total load. If we evaluate the similar safety factor for the total load (including dead load) $s_t = p_{t,ult}/p_{t,ser} = 1.99$, which corresponds to usual safety level.

5 CONCLUDING REMARKS

Non-linear finite element analysis based on advanced constitutive models can be well used for the simulation of a real behaviour of reinforced concrete structures. Computer simulation is a relatively new and robust tool for checking the performance of concrete structures in design and development. Such simulation can be regarded as virtual testing and can be used to confirm and support the structural solutions with complex details or non-traditional problems and can serve to find an optimal and cost-effective design solution. Simulation is useful in such cases, which are not well covered by the code of practice provisions. An interesting application is also in assessment of the remaining structural capacity of existing structures and investigating the causes of damage and failures. It can support the creativity of engineers and contribute to safety and economy of designed structures.

ACKNOWLEDGEMENT

This paper is based in part on the research projects supported by the Czech Grant Agency no. 103/00/1563 and 103/99/0755. The financial support of this research by the Czech Grant Agency is greatly appreciated.

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