Nonlinear Analyses for the Design of Safety-Critical Concrete Structures

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Abstract. Nonlinear analysis and simulation using finite element analysis can provide engineers with an insight into the real structural performance and behavior. Contrary to the traditional design approached based on the elastic beam theory, the nonlinear models can evaluate complex 3D stress states within the material and simulate real material behavior, including crushing of concrete in compression, cracking in tension or yielding of the steel reinforcement as well as long-term rheological phenomena. Design guidelines and suitable safety formats for nonlinear analysis are becoming available in the new design codes such as the latest fib Model Code 2020 and the new generation of Eurocodes. An important aspect of the application of nonlinear analysis and simulation in design is the consistent treatment of model uncertainties. The most consistent treatment of suitable safety formats for nonlinear analysis is available in the fib model code, where several methods are proposed based on the partial safety factor method, global resistance, and full probabilistic methods. These safety formats will be briefly presented in the paper. Two case studies from the engineering practice from Europe and the US will be presented to demonstrate the feasibility and advantages of nonlinear analysis in the design of new and assessment of existing bridges. First, a pre-stressed segmental concrete bridge, where excessive shear cracks developed in the girder web during the construction process, will be presented. The nonlinear FE analysis was used to investigate and confirm the origin of the cracking and, subsequently, it was used to design and simulate the effect of additional strengthening. The final example shows an application of an advanced chemo-mechanical model for durability prediction of a reinforced concrete bridge, where chloride-induced reinforcement corrosion is simulated to investigate how it affects the load-bearing capacity during the service life.

INTRODUCTION

The nonlinear FEM simulation and analysis is starting to be more often used by engineers as an ideal tool for checking the design and behavior of critical structures or structural elements. One of the main advantages of nonlinear modelling is that it can provide very useful insight into the real behavior of structures, and it helps to determine its failure mechanism and discover possible critical weak points. For the application in engineering practice, proper guidelines need to be available. Currently, these provisions are given in the fib Model Code 2010 [1] and will be introduced in the new generation of Eurocodes. An important aspect of these standards is the introduction of the model uncertainty, which should be properly analyzed and calibrated, and in most cases needs to be evaluated specifically for each material model or software package.

This paper provides first a brief theoretical overview of the nonlinear FEM. An interesting insight into the model uncertainty is obtained by studying examples from benchmark competitions, and finally presenting a code-based framework for engineering application.

The second part of the paper then shows two examples from practice, where the nonlinear analysis was used to assess the performance of a post-tensioned reinforced concrete bridge and for a digital twin approach in the analysis of the long-term durability behavior of bridges.

NONLINEAR ANALYSIS OVERVIEW

Before presenting the selected examples of application, the framework for nonlinear analysis is briefly described. We briefly cover the theory, show validation against experimental data, and explain the uncertainties in the modeling and the safety format for the application of the nonlinear FEM in engineering practice.

The essential part of the nonlinear finite element analysis is material models that can realistically describe the behavior of brittle cementitious material such as concrete. In the field of material science, this is most commonly represented by a stress-strain constitutive relationship. The material model should respect the physics principles and in the case of brittle materials should properly take into account the energy dissipated during the damage processes and volumetric dilation during concrete crushing.

The analyses presented in this paper were calculated using the ATENA software package, which implements the fracture-plastic model proposed by Červenka J. et al. [2,3,4]. It divides the nonlinear material response into tension and compression.

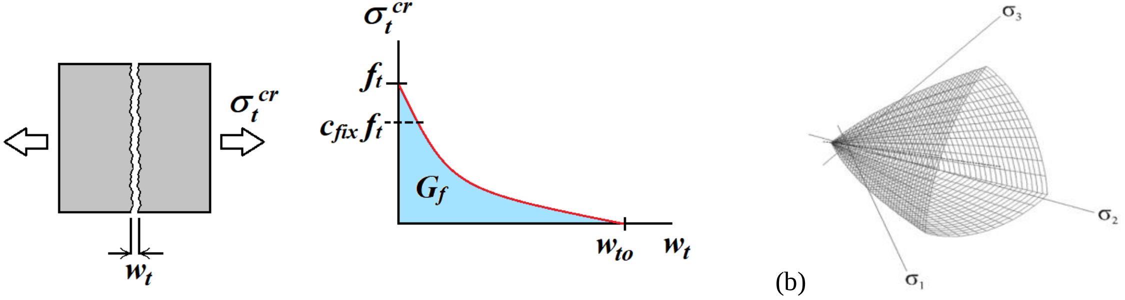


FIGURE 1. *The fracture crack opening law that controls the softening response in tension (a) and 3D plasticity criterion for concrete crushing*

The tensile post-peak response is characterized by an orthotropic smeared crack model with a softening curve controlled by the fracture energy that is dissipated during the crack propagation as shown in Fig. 1. Rather than explicitly tracking each individual crack, the smeared crack approach adds the response of multiple cracks within a single element and adequately modifies the strength and energy dissipation of the element. The cracking model is orthotropic and allows the formation of up to three cracks in the three principal directions.

It has been observed that the smeared crack models suffer certain mesh dependency. For instance, if large elements in order of hundreds of millimeters or even meters are used in the model, the assumption that a single crack develops in a principle tension direction is no longer valid. In reality, several cracks form in the case of a reinforced concrete sample. Therefore, the total fracture energy available for dissipation is underestimated in the simulation thus reducing the peak load. This can be adjusted by an additional material parameter specifying the crack spacing [5]. Analogically, if a very small mesh is used, the number of cracks may be overestimated. In reality, the minimum crack spacing would be limited by the internal material length scale depending on the aggregate size [5]. By imposing a limit on minimum crack spacing distance, it can be ensured that the crack will localize in a physically plausible distance range.

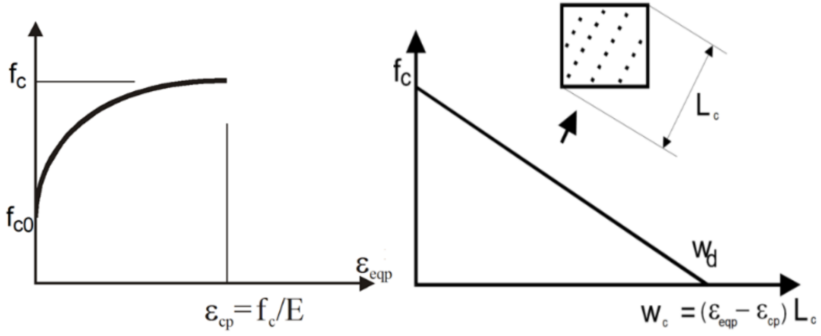


FIGURE 2. *Hardening/softening diagrams for the plasticity model for concrete in compression.*

The compression branch is described by the plasticity approach with the Menetrey & Willam failure criterion [6] shown in Figure 2. The figure shows the hardening elliptical curve after exceeding the stress level corresponding to the onset of crushing f_{c0} and linear softening after reaching the compressive strength. The material model incorporates a yield surface (Fig. 1) and non-associated flow rule to capture the plastic strain evolution during the concrete crushing.

When nonlinear material laws are introduced into the FEM, the set of equations to be solved becomes nonlinear. Therefore, a suitable solver technique is necessary to find the equilibrium between the nodal displacement and material response. Most commonly, these methods are derived from the well-known Newton-Raphson method. The iterative solution runs until the residual error decreases below the prescribed convergence criteria. It is needless to say that only the results, where the convergence of the solution was reached, should be used for structural analysis. The loss of convergence is sometimes an indicator that the ultimate load-bearing capacity was exceeded; however, the results should be always carefully inspected to determine the actual cause of the divergence.

Once the convergence at a given load step is obtained, the next load step is calculated based on the previously calculated state. Unlike in the linear (i.e., elastic) solution, the superposition principle is not valid, meaning that the structural response under multiple loadings cannot be found by simple addition. Therefore, the loading history plays an important role in the simulation and should resemble the actual loading scenario.

Most engineering applications are formulated as load-prescribed tasks since the design standards generally specify the external loads. For this purpose, the arc-length method [7,8] is more suitable as it scales the load vector based on the displacement increment. Thus the applied load is automatically scaled down when maximum load-carrying capacity is reached. The arc-length method allows tracing the structural response when the ultimate load-carrying capacity is reached into the post-peak behavior.

VALIDATION AND BENCHMARK PREDICTIONS

The nonlinear numerical models and methods must be validated against experimental data. This is often done using experiments with known results, however, the most interesting type of validation is the blind predictions. They are often organized as blind contests aiming at structural types or failure mechanisms that are difficult to predict and still not fully understood. For instance, the bending failure can be simulated with better confidence than the shear or punching failure mechanism. Similarly, blind tests are conducted also for new materials such as fiber-reinforced concrete.

The team of the author participated in the past in many blind competitions involving mainly shear and punching types of failures, which are usually considered more difficult to predict. A more detailed summary of these competitions with material and geometric data can be found in the publication [9]. In this paper, only a schematic description of these blind competitions is provided in Figure 3.

Figure 4 shows the accuracy of the blind predictions of the author’s team over a large range of competitions spanning almost 40 decades. The figure presents the accuracy of the strength predictions of the author’s team highlighted in green as well as the overall span of results from the other participants. For the TU Delft contest [15], the range of other predictions is not yet available as there is no official publication of the contest results yet. Overall, it is possible to conclude that the quality of numerical predictions is quite satisfactory. In the case of higher errors, it was mostly on the conservative side. The only high overestimations of the strength appeared in fib 2020 [12] and in the first test of the TU Delft 2023 [15] contest.

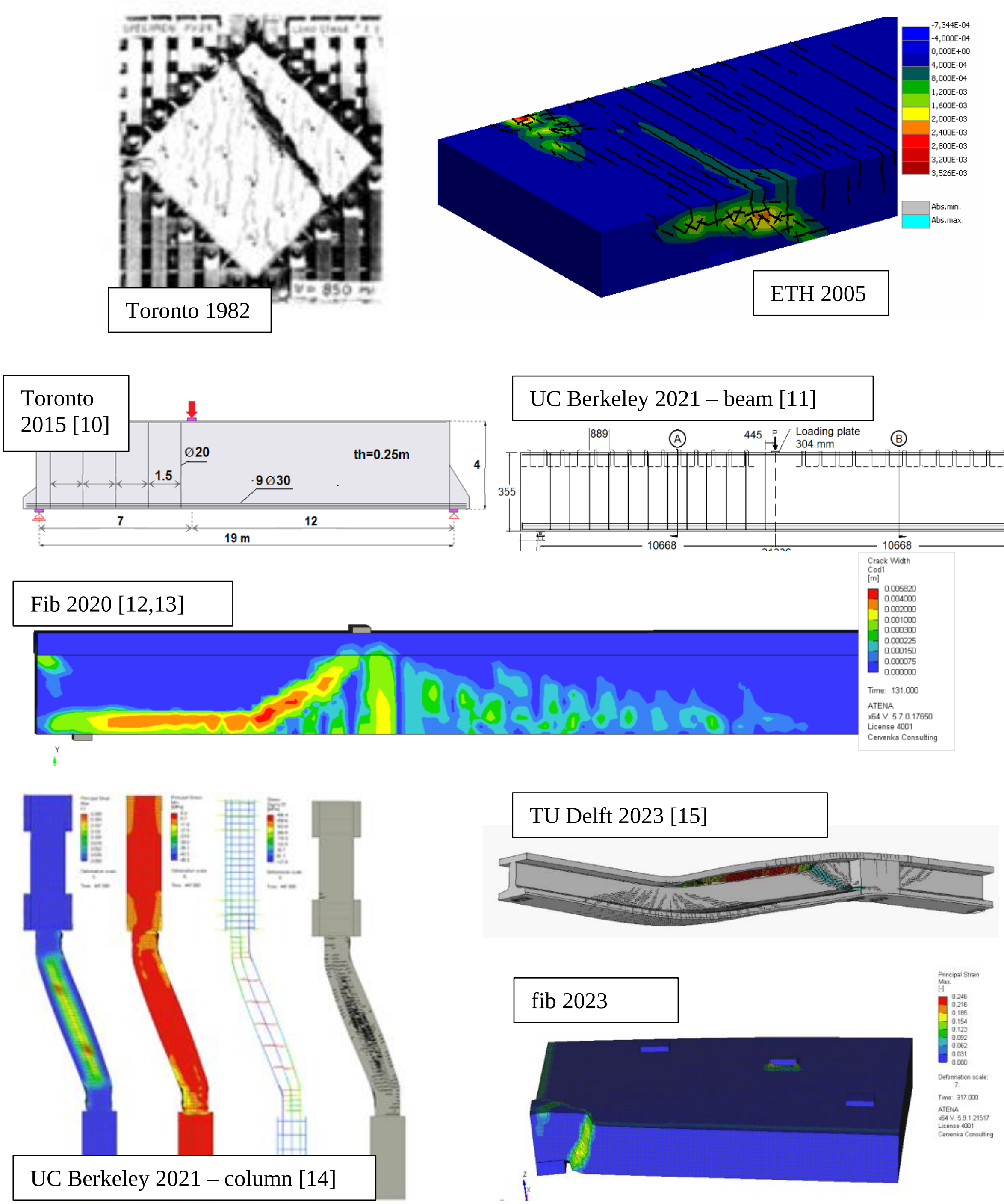
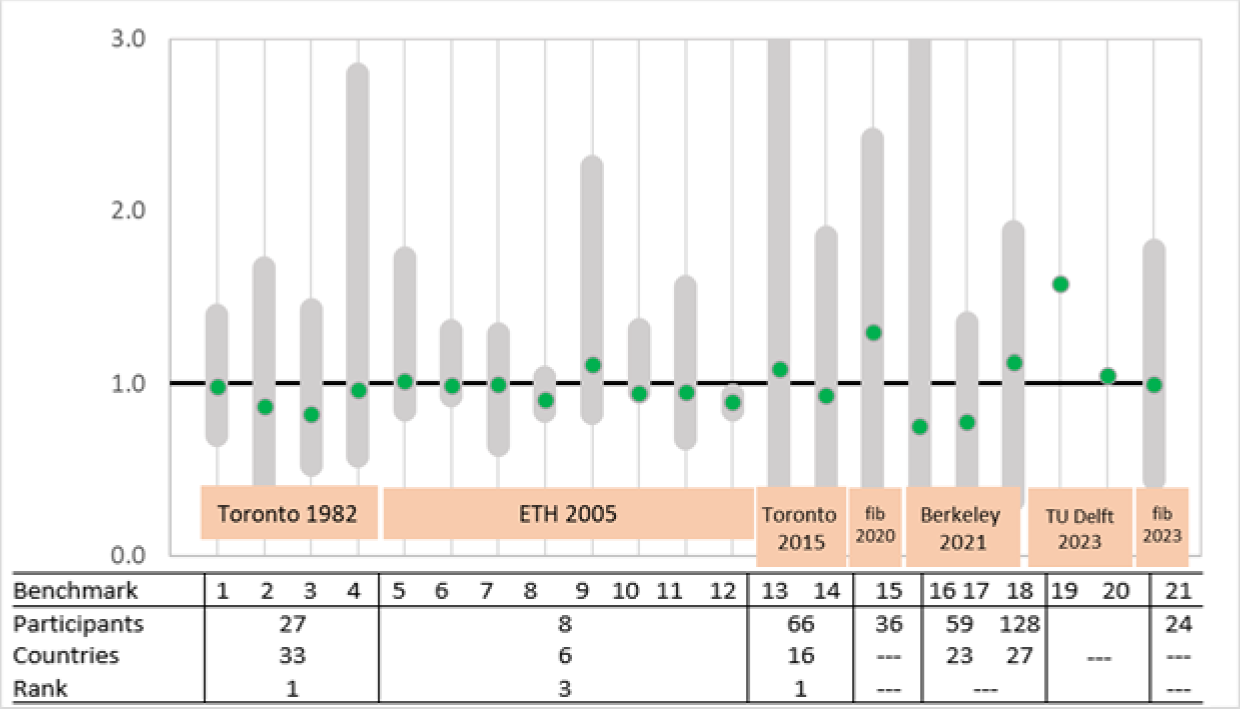


FIGURE 3. *Overview of benchmark problems [10,11,12,13,14, 15]*

Fib 2020 [12] competition involved a fiber-reinforced concrete (FRC) beam failing in shear, which is a rather novel material with not sufficient knowledge, especially on the shear behavior. The experience from this contest was analyzed in the publication [13], which emphasized the importance of the proper model calibration using the lower bound of the material tests of FRC material. This knowledge was successfully applied in the second fib 2023 test involving a punching failure of FRC, where a perfect match with the test strength was obtained. The overestimation of the first test in TU Delft 2023 [15] can be attributed mainly to a modelling error, where the interface between the top beam flange and the bottom prefabricated inverted I beam was not included in the model.

The blind prediction competitions provide a very interesting view on the uncertainties involved in predicting the behavior of reinforced concrete structural elements and show that proper understanding is still generally not available. However, they typically suffer from several major drawbacks. The material uncertainty is typically not considered, with the exception of the fib 2020 and 2023 competition, where at least two tests were always performed, and at least basic statistical properties were available. The second problem, which is typical for most blind competitions, is the availability of only basic material parameters for concrete. Typically, only concrete compressive strength is available, while for accurate modelling of concrete structures at least tensile strength and fracture energy would be necessary. The only exception is the FRC blind competitions organized by Prof. Barros, ie. fib 2020, 2023, where some basic material tests were available for the calibration of the material model parameters.



Benchmark	1 2 3 4	5 6 7 8 9 10 11 12	13 14	15	16 17 18	19 20	21
Participants	27	8	66	36	59 128		24
Countries	33	6	16	---	23 27	---	---
Rank	1	3	1	---	---		---

Figure 4. *Summary of results from several competitions*

Safety Format for Nonlinear Analysis

For the application of nonlinear FE analysis in engineering practice, an appropriate safety framework needs to be available. The structural requirements specify that the design structural resistance R_d must be greater than the effect of design loads E_d. Therefore:

$$E_\mathrm{d} < R_\mathrm{d} = \frac{R_\mathrm{d}^\mathrm{FE}}{\gamma_\mathrm{Rd}} \tag{1}$$

The fib Model Code [1] defines three kinds of nonlinear methods for obtaining the design structural resistance. These are the full probabilistic, global resistance, and partial factor methods (PFM).

The method closest to the traditional approach in cross-sectional design is the PFM. It specifies that the material parameters used in the nonlinear analysis are derived from the design values of the concrete compressive strength for concrete and reinforcement design yield strength or rapture strain. During the nonlinear simulation, the design load combination is gradually increased until the maximum load-bearing capacity $R_\mathrm{d}^\mathrm{FE}$is found. The maximum load value gives the global resistance R_d^FE, which should be further reduced by the model uncertainty partial safety factor γ_Rd to obtain the structural design resistance.

Another semiprobabilistic global resistance approach is the estimate of the coefficient of variation (ECoV) originally proposed by Červenka V. [18]. It assumes that the design structural resistance follows the lognormal distribution, which can be characterized by the characteristic R_k and mean structural R_m resistances. From these, the coefficient of variation V_R can be estimated as:

$$V_{\mathrm{R}} = \frac{1}{1.65}\, ln\left(\frac{R_{\mathrm{m}}}{R_{\mathrm{k}}}\right) \tag{2}$$

and the global resistance factor γ_R is calculated using the assumption of lognormal distribution:

$$\gamma_{\mathrm{R}} = exp(\alpha_{\mathrm{R}}\, \beta\, V_{\mathrm{R}}) \tag{3}$$

where α_R is the sensitivity factor for resistance in MC 2010 and EC2 with a recommended value of 0.8 for a 50-year reference period. β is the target value for the reliability index typically 3.8 in MC and EC2 for a 50-year reference period. The design structural resistance according to the ECoV method is calculated:

$$R_{\mathrm{d,ECOV}} = \frac{R_{\mathrm{m}}}{\gamma_{\mathrm{R}}\, \gamma_{\mathrm{Rd}}} \tag{4}$$

The fib Model Code [1] also lists the full probabilistic method; however, this method will be quite demanding for typical engineering applications as it often requires hundreds of nonlinear analyses.

MODEL UNCERTAINTIES AND SAFETY FACTORS

Model uncertainty is generally described as the ratio of the resistance found experimentally R_{exp} and the resistance obtained in the simulation R_{sim}:

$$\theta = \frac{R_{\mathrm{exp}}}{R_{\mathrm{sim}}} \tag{5}$$

It can be considered as a random variable that can be obtained by statistical evaluation of simulation results of various experiments. Assuming the lognormal distribution of the evaluated dataset, the safety factor for model uncertainty γ_{Rd} can be calculated as:

$$\gamma_{\mathrm{Rd}} = \frac{\exp(\alpha_{\mathrm{R}} \cdot \beta \cdot V_{\theta})}{\mu_{\theta}} \tag{6}$$

where μ_θ is the mean value of the model uncertainty and V_θ is the coefficient of variation from the model uncertainty calculation. For the sensitivity factor for the reliability of resistance α_R and the reliability index β, values of 0.8 and 3.8 can be again taken from [1].

It should be understood that the model partial safety factor is specific to a given software package or material model. Furthermore, it can differ based on the failure mode, i.e. bending, shear, or compression. For the ATENA software package [2] with the fracture-plastic material model [4], the model uncertainty is evaluated in the publication [19] for 33 typical cases of reinforced concrete structural elements with failure modes ranging from bending, shear and punching failure mechanisms.

Table 1. *Recommended model uncertainty partial factor for ATENA software [19]*

Failure Type	μ_θ	V_θ	γ_{Rd}
Punching	0.971	0.076	1.16
Shear	0.984	0.067	1.13
Bending	1.072	0.052	1.01
All failure modes	0.979	0.081	1.16

Similar model uncertainty studies have been performed for other models and finite element software codes by other researchers such as for instance Engen [20], Castaldo [21,22] and Gino [23]. The obtained uncertainty factors were mostly in the range 1.02 – 1.19 except for the study [22], which included also cyclic load cases, and the model uncertainty factor 1.35 was obtained.

EXAMPLES OF APPLICATION

Strengthening of box girder bridge

The first example of the application of the nonlinear FEM shows the analysis of a viaduct in the Czech Republic. The objective of the analysis was the assessment of the cause of diagonal shear crack development occurring soon after the bridge construction as well as the assessment of proposed strengthening measures. The nonlinear full 3D analysis showed the limits of the standard approach using linear beam elements. For large shear stresses that developed due to thermal loads during the construction, the assumption of the planar beam cross-section is not necessarily valid, as was demonstrated by the 3D nonlinear model. Furthermore, the nonlinear analysis also considers the redistribution of the internal forces due to cracking. A brief summary of the analysis is given here.

Figure 5. *A view of the investigated viaduct.*

The bridge is an integral part of the main highway connecting the Czech capital Prague with the Saxony region, Germany. From the structural viewpoint, it is a post-tensioned reinforced concrete box girder bridge with a span of 48 meters. The depth of the box girder is 2.7 meters. The bridge is shown in Fig. 5. During construction interruption, diagonal cracks formed in the web of the box girder at the segments close to the pier supports (Fig. 5 right).

In the nonlinear analysis, a typical section of the bridge corresponding to the pier and half of the span on each side was modeled. At the midspans, the internal forces obtained from a global beam model were applied as the external load to ensure the realistic behavior of the partial model.

The analysis considered the actual history of the structure, including the construction phase. The construction process among others simulated the balanced cantilever construction method, construction interruption, and high thermal loads during this period. The compliance function from Eurocode [24] was used for considering the long-term concrete creep. The observed crack pattern was successfully reproduced in the simulation. The main crack propagation was associated with the large thermal gradient that developed during extremely hot summer days during the construction break for almost 6 years.

In response to the shear crack formation suggesting a possible weak point, the operating authorities decided to implement strengthening measures. The strengthening should ensure that the cracks will not propagate further and enhance the overall robustness. It was designed in the form of additional post-tensioned cables placed in the box girders and anchored by special steel deviators.

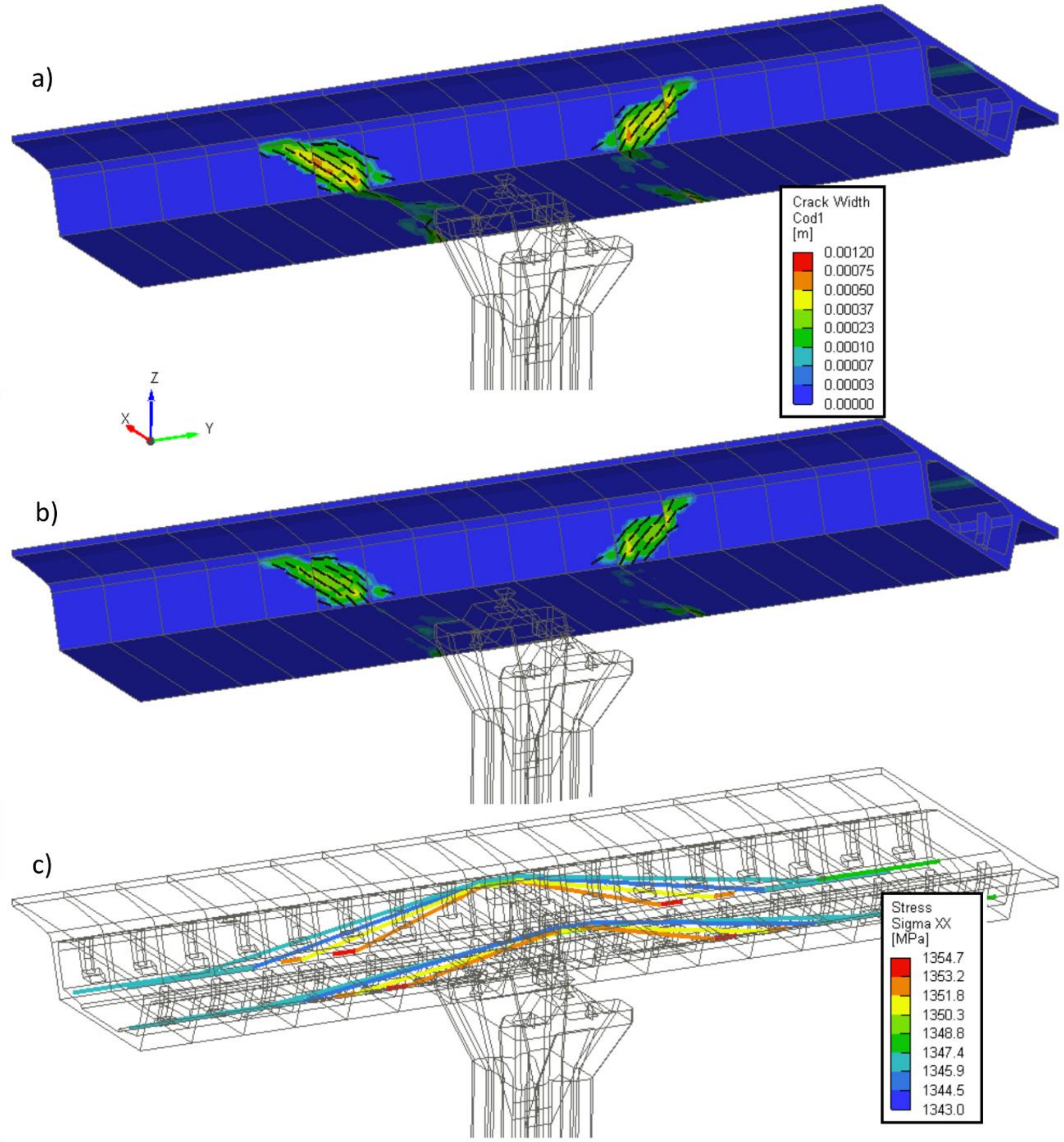


Figure 6. *Results from the nonlinear analysis of the viaduct strengthening: a) crack pattern and width before strengthening, b) after strengthening, and c) arrangement of strengthening cable and their stresses, (only cracks larger than 0.1 mm are shown).*

Figure 6 shows the main analysis results. Before the strengthening, the maximum crack width in the box girder web was in the range of 0.50 - 0.75 mm (Fig. 9a). This crack width corresponded to the moment when the construction process was finished and the bridge was opened for traffic. After the strengthening cables were installed, the crack width decreased to 0.20 - 0.40 mm (Fig. 9b).

Reinforced Concrete Bridge in Germany

The other example demonstrates the application of nonlinear modelling in the Digital Twin approach applied to a bridge in Germany. The Vogelsang Bridge in Germany consists of eight partial structures built in three different construction types. The bridge was built between the years of 1971 and 1973. The total length is approximately 595 m and it has a total area of 9 744 m² including ramps. For the monitoring, two spans of 13.8 + 13.2 m were chosen. From the structural point of view, this section is a continuous non prestressed slab with the height of 0.6 m. The bridge monitoring ran for 61 days from Jan. until Mar. 2019.

Figure 7. *Digital Twin monitoring application at Vogelsang bridge in Germany.*

The Digital Twin approach consisted of the iBWIM (Bridge-Weigh-In-Motion) technology (PEC – Petschacher Consulting, ZT-GmbH) monitoring system, which was coupled with the nonlinear model to facilitate its calibration and validation.

The system consisted of strain measurement units coupled with a laser rangefinder, which is used for vehicle detection (see Fig. 7 top). The measurement units are mounted on the underside of the bridge; therefore, the traffic is not interrupted during the installation. Each unit consists of strain gauges and a data collector. The strain gauges are arranged both in the transverse and longitudinal directions.

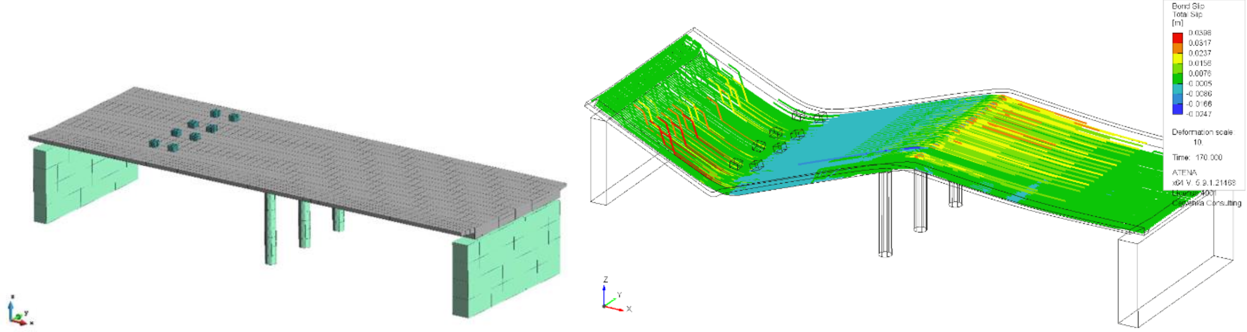


Figure 8. *The numerical model for the Digital Twin of the Vogelsang bridge (left), reinforcement bond failure at peak load in the nonlinear long-term durability analysis (right).*

The monitoring system was calibrated by crossing the bridge with trucks of known weights. The information from the coupled system of the strain gauges and the laser range-finder can be used for rather unique analyses of the bridge traffic. It is possible to collect data on the bridge response as well as the vehicle speed, weight, and load distribution over the vehicle's axles. The sensitivity of the measurement system is tuned to detect vehicles of a gross weight above 3.5 t.

The calibrated numerical model is used for long-term predictions of bridge deterioration due to reinforcement corrosion. The chloride ingress and reinforcement corrosion models applied in the analysis assume two phases of the process. First, during the induction phase, the chlorides penetrate the concrete microstructure. This process is often described by the diffusion equation [25]. The rate of chloride ingress depends both on the material behavior through the chloride diffusion coefficient and the chloride-binding ability, and the structure's exposure conditions determined by suitable boundary conditions. The model takes into account the presence of cracks from mechanical loads, which accelerate the chloride ingress. Once the chloride concentration around the reinforcement reaches the critical level, the corrosion is initiated, and the propagation phase begins.

At first, the rate of the corrosion process is driven by the chloride concentration, temperature, corrosion time, and, through the pitting factor modelling the corrosion localization. As the corrosion proceeds, it is assumed that only the uncorroded portion of the reinforcement cross-section transfers mechanical stresses. Since the corrosion products have a larger volume than the steel, internal pressure builds up in the concrete cover. The implemented model assumes that once the critical corrosion depth is reached, spalling of the concrete cover occurs. This critical corrosion depth depends on the concrete strength, the initial reinforcement diameter and the concrete cover. The loss of concrete cover due to spalling is not modelled directly, but only indirectly by increasing the rate of corrosion on the basis of the structure's exposure conditions.

During the long-term corrosion process, the numerical model is loaded up to failure at various aging times to evaluate the bridge load-carrying capacity degradation. These results are plotted in Fig. 9, and they represent the most interesting result from the durability analysis. This information can be used for the optimization of the bridge maintenance and for efficient planning of future rehabilitation or retrofitting.

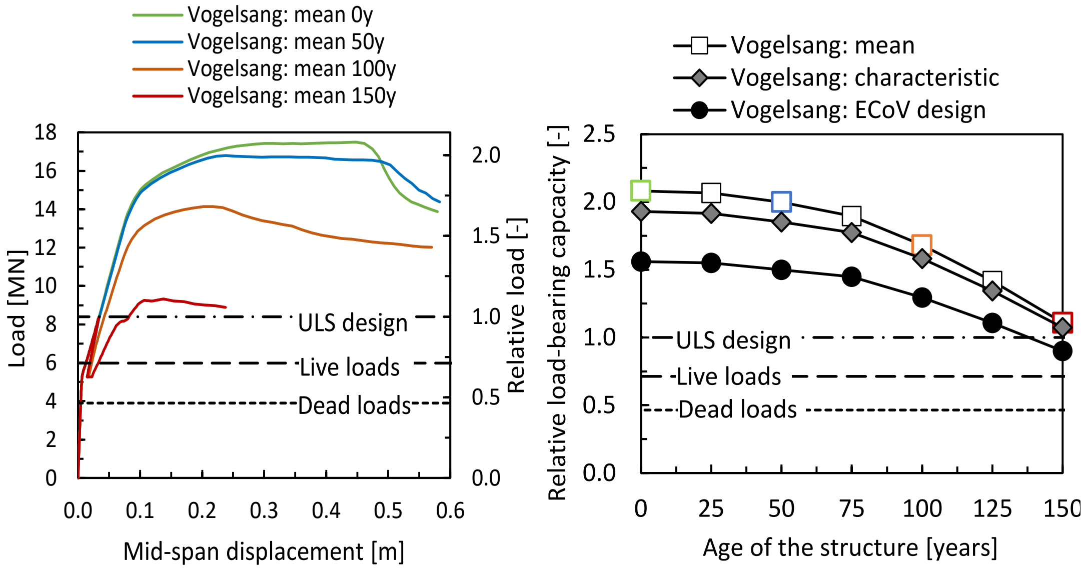


Figure 9. *Load-displacement curves for the Vogelsang bridge (left) and the evolution of bridge resistance in time due to reinforcement corrosion (right).*

SUMMARY

This paper presents the fundamentals for the application of nonlinear analysis for the design and assessment of reinforced concrete structures and bridges. The most important component of a nonlinear numerical simulation is the material model that can realistically simulate the performance of the real material, including the failure (i.e., post-peak) response. For typical engineering applications, the smeared crack model with a crack band is available and has been proven to accurately reproduce laboratory tests as well as blind predictions. The accuracy of a given material model and software package is described by a model uncertainty and reflected by the model safety factor.

In the second part of this paper, application examples are presented. The first example shows an assessment of the pre-stressed segmental bridge with detected diagonal shear cracks. The nonlinear analysis helped to explain the origin of the cracks that could not be satisfactorily explained by standard methods based on elastic beam analysis. The assessment of the bridge strengthening was performed using the new safety formats for nonlinear analysis [1].

The last example demonstrates the unique application of nonlinear analysis in the modern concept of Digital Twins, where the calibrated nonlinear model provides unique predictions of the bridge durability that can be used for efficient and sustainable maintenance of existing infrastructures.

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REFERENCES

1. International Federation for Structural Concrete. *fib Model Code for Concrete Structures 2010*. 2013. 434 p.
2. Červenka V, Jendele L, Červenka J. *ATENA Program Documentation: Part 1 Theory*. Cervenka Consulting s.r.o., Prague; 2022. 360 p.
3. Červenka J, Červenka V, Eligehausen R. Fracture-plastic material model for concrete, application to analysis of powder actuated anchors. In: *Proceedings FRAMCOS (3)*. 1998. p. 1107–16.
4. Červenka J, Papanikolaou VK. Three dimensional combined fracture–plastic material model for concrete. *International Journal of Plasticity*. 2008 Dec 1;24(12):2192–220.
5. Červenka J, Červenka V, Laserna S. On crack band model in finite element analysis of concrete fracture in engineering practice. *Engineering Fracture Mechanics*. 2018;197:27–47.
6. Willam PM and KJ. Triaxial Failure Criterion for Concrete and its Generalization. *ACI Struct J*. 92(3).
7. Crisfield MA. An arc-length method including line searches and accelerations. *Int. J. Numer. Methods Eng*. 1983 Sep 1;19(9):1269–89.
8. Červenka J, Šmejkal F, Červenka V, Kurmann D. On the application of nonlinear analysis in the design and assessment of reinforced concrete structures. In: Meschke, Pichler, Rots, editors. *Computational Modelling of Concrete and Concrete Structures*. CRC Press; 2022. p. 8–21.
9. Červenka J., Červenka V., Skandis U., Řehounek L., Uncertainty in the simulation of concrete fracture and comparison with blind competitions, *11th International Conference on Fracture Mechanics of Concrete and Concrete Structures*, proceedings FraMCoS-11 conf., J. M. Chandra Kishen, A. Ramaswamy, S. Ray and R. Vidyasagar (Eds), 2023
10. Collins MP, Bentz EC, Quach PT, Proestos GT. The challenge of predicting the shear strength of very thick slabs. *Concrete International*. 2015;37(11):29–37.
11. Moehle J, Zhai J. Thick Foundation Element Blind Prediction Contest [Internet]. 2021. Available from: peer.berkeley.edu/news-and-events/2021-thick-foundation-element-blind-prediction-contest
12. Barros, et al. 2020, Blind competition on the numerical simulation of steel-fiber-reinforced concrete beams failing in shear, *Struct. Concrete, Journal of the fib*, 2020, https://doi.org/10.1002/suco.202000345
13. Skadins U., Cervenka, J., Lessons learnt from blind competition of shear behavior of fiber reinforced concrete T-beam, *Struct. Concrete, Journal of the fib*, 2023, https://doi.org/10.1002/suco.202201187
14. Gunay, S., Donald, E., 2021. Reinforced Concrete Column Blind Prediction Contest, https://peer.berkeley.edu/news-and-events/2021-reinforced-concrete-column-blind-prediction-contest
15. Ibrahim, M.S., Kostense, N.W., Poliotti, M., Yang, Y., 2023, TU-Delft blind prediction contest on prestressed concrete beams made continuous, https://concrete-prediction-contest.tudelft.nl
16. European Committee for Standardization. Eurocode 2: Design of concrete structures - Part 1-1: General rules, and rules for buildings. prEN 1992-1-1:2020. 2020. proposal draft, CEN/TC250/ SC2/WG1, document N 1057
17. Červenka V. Global Safety Format for Nonlinear Calculation of Rein-forced Concrete. *Beton- und Stahlbetonbau*. 2018;103:37–42.
18. Červenka, V. 2013. Reliability-based non-linear analysis ac-cording to Model Code 2010, Journal of fib, Structural Concrete 01/2013, pp.19-28
19. Červenka V, Červenka J, Kadlec L. Model uncertainties in numerical simulations of reinforced concrete structures. *Structural Concrete*. 2018;19(6):2004–16.
20. Engen M, Hendriks MAN, Köhler J, Øverli JA, Åldstedt E. A quantification of the modelling uncertainty of non-linear finite element analyses of large concrete structures. *Structural Safety*. 2017 Jan 1;64:1–8.
21. Castaldo P, Gino D, Bertagnoli G, Mancini G. Partial safety factor for resistance model uncertainties in 2D non-linear finite element analysis of reinforced concrete structures. *Engineering Struct.* 2018 Dec 1;176:746–62.
22. Gino D, Castaldo P, Giordan L, Mancini G. Model uncertainty in non-linear numerical analyses of slender reinforced concrete members. *Structural Concrete*. 2021;22:845– 870.
23. Castaldo P, Gino D, Bertagnoli G, Mancini G. Resistance model uncertainty in non-linear finite element analyses of cyclically loaded reinforced concrete systems. *Eng Struct*. 2020 May 15;211:110496.
24. European Committee for Standardization. *Eurocode 2: Design of concrete structures - Part 1-1: General rules, and rules for buildings*. 2006.
25. The European Union–Brite EuRam III (2000) *Probabilistic performance based durability design of concrete structures*: Final technical report of Duracrete project