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ARTICLE

Comparison of advanced semi-probabilistic methods for design and assessment of concrete structures

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Abstract

This paper presents the comparison of advanced semi-probabilistic methods for the design and assessment of concrete structures represented by mathematical models solved by non-linear finite element methods. The special attention is given to the advanced methods focused on the estimation of the coefficient of variation of structural resistance. Numerical examples represent a replication of laboratory experiments of beams with different failure modes. The obtained results are discussed with respect to the accuracy of the employed methods and the influence of the assumed statistical correlation among basic variables. Simplified methods give a good estimation of the design values, though their accuracy is dependent on the type of the failure mechanism. Moreover, it is shown that mutual correlations among random variables may significantly affect the design value of resistance, and they should be carefully defined and modeled.

KEYWORDS

non-linear finite element method, reliability of concrete structures, safety formats, semi-probabilistic methods, statistical analysis, Taylor series expansion

Abbreviations: R, structural resistance; E, action effect; F^{-1}, inverse cumulative distribution function; α, sensitivity factor; β, target reliability index; R_d, design value of resistance; γ_M, partial safety factor for a material property including model uncertainty; γ_m, partial safety factor for a material property without model uncertainty; γ_{R_d}, model uncertainty in γ_M; v_X, CoV of resistance variable X in PSF; μ_X, mean of resistance variable X in PSF; d, effective depth; f_y, yield strength of reinforcement; f_c, compressive strength of concrete; A_c, concrete area; η, conversion factor; θ, model uncertainty; f_{ct}, tensile strength; G_f, fracture energy; R_m, simulation with mean values of basic variables; R_k, simulation with characteristic values; X_k, characteristic values (5% quantile); X_m, mean values of basic variables; X_k^*, characteristic values including uncertainty according to PSF; $r(\mathbf{X})$, the function of structural resistance; $\mathbf{X}$, input random vector of N basic variables; μ, mean value; v, coefficient of variation; c, step size parameter; Θ, 1D eigen distribution; σ^2, variance; λ_1, the first eigenvalue of input covariance matrix; $X_{i\Delta}$, reduced basic variable for Eigen ECoV; ρ, reinforcement ratio; Δ_Θ, distance between μ_Θ and desired quantile $F_\Theta^{-1}(\Phi(-c))$; Φ, cumulative distribution function of the standardized normal distribution.

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1 | INTRODUCTION

The determination of a design value of resistance ensuring a target level of structural reliability represents a key task for engineers. Design codes like Eurocodes offer a clear background for that, making it possible to separate structural resistance and action effect in most design situations. A relative importance of resistance and of the load effect is expressed through the respective sensitivity factors whose recommended values commonly ensure reasonably conservative design solutions. As structural analysis can then be fully focused on resistance, the approach is called semi-probabilistic.

Various procedures to determine the design value of resistance are generally termed as safety formats. They are elaborated at different levels of simplification and accuracy. Safety formats according to Eurocodes fully rely on the partial factor method; also the EN 1992-2:2005[1] approach belongs to this category. The design value of resistance is obtained by one calculation of the computational structural model using the design values of basic variables. While this approach performs well and is fully justified for linear computational models, its use for non-linear models is questionable and can lead to an over-conservative design resistance[2,3] or may even fail.[4] In contrast, it is well known that the probabilistic analysis in combination with a non-linear finite element analysis offers a significant added value over the standard linear analysis and semi-probabilistic approach implemented in Eurocodes,[5] and researchers are highly motivated to develop novel techniques coupling the accuracy of the non-linear finite element analysis (NLFEA) with a realistic description of the basic variables by probabilistic modeling.

The only general tool for probabilistic analysis is represented by the Monte Carlo simulation (MC), simulating uncertainties with their complete probability distribution and statistical correlation. For a large number of simulations, the approach leads to the complete information about the distribution of resistance, but the number of simulations is often limited, and the design value of resistance is estimated based on the estimates of the mean value and coefficient of variation (CoV). The accuracy depends on the quality of these estimates. Even if the advanced stratified sampling such as Latin Hypercube Sampling (LHS)[6] is used, a number of simulations may range from tens to hundreds. There are more advanced and computationally demanding MC techniques,[7] but their implementation is usually far more complicated, and thus they are typically utilized for scientific applications only. The computational burden of MC represents the main obstacle of the approach for time-consuming mathematical models like NLFEA, since it is not computationally feasible for industrial applications. Significant efforts have been made to reduce the computational cost of the estimation of statistical moments and its dependency on number of input random variables. Recently, promising results were obtained by high-dimensional model representation method[8] and its later modifications, such as the Maximum Entropy Multiplicative Dimensional Reduction Method.[9] Although these methods represent a significant improvement for general estimating of statistical moments, it is still necessary to perform tens of numerical simulations.

That is why alternative techniques focused on the Estimation of Coefficient of Variation (ECoV) of structural resistance have been developed. They represent a compromise between the simple and, in most cases, the conservative approach of partial factors and MC. They consider uncertainties in the form of N basic (input) random variables, but under several simplifying assumptions, they reduce the computational model calculations to a very low number acceptable in practice:

- ECoV according to *fib* Model Code 2010[10]—2 numerical calculations for any N,
- Eigen ECoV[11]—3 numerical calculations for any N,
- ECoV based on Taylor Series Expansion[12,13]—$N + 1$ or $2\,N + 1$ numerical calculations, i.e. only three calculations when the concrete compressive strength and the yield strength of the reinforcement are modeled as stochastic variables for the whole structure, but an excessively increasing number when random material properties are modeled at various locations at the structure, or when the stochastic model contains additional random parameters.

The ECoV methods commonly simplify an estimation of the mean value of resistance as a result of the calculation of the computational model using the mean values of input variables. This assumption is strong (particularly for highly nonlinear models), though it can be accepted in many applications where ECoV methods achieve sufficient accuracy.[14–17] The second strong simplification common for all ECoV methods is assuming a lognormal distribution of resistance.

Note that LHS can be used as an ECoV technique for the estimation of the mean value and CoV.[17] In particular, for a low number of simulations (tens), the assumption of lognormal distribution is necessary, as a reliable estimation of distribution of resistance generally requires a higher number of simulations. A similar situation applies to the methods based on numerical quadrature,[18] however, they are extremely computationally expensive for increasing N, and thus are rarely employed in industry.

This paper presents a summary of the available approaches and the comparison of the semi-probabilistic methods for practical examples of NLFEA. In contrast to the review of ECoV methods,[19] a special attention is given to the verification of the recently proposed adapted Taylor series expansion (TSE), and a special case of TSE referenced as Eigen ECoV, which does not bring a significant additional computational burden, but extends the range of applicability of ECoV according to *fib* Model Code 2010.

The aim of the paper is to contribute to the discussion and clarify the recommendations provided by the nearly complete drafts of fib MC 2020 and prEN 1992-1-1.[20] The general theoretical background of the semi-probabilistic approach, PSF, and selected ECoV methods are described in Section 2. Note that there is a strong reasoning for the selection of these particular simplified ECoV methods, since they are based on the general formulation of TSE. The purpose of this section is thus not only to summarize the selected methods, but also to clarify their similarities and assumed simplifications. Besides the simplified ECoV methods and PSF, a brief summary of stratified sampling is also presented, since we use this method for a reference solution of the numerical examples. Section 3 describes selected case studies and the methodology of the numerical examples, i.e. our strategy for a proper comparison of the selected methods. Besides the comparison of the ECoV methods, we also present their theoretical characteristics regarding the correlation among input random variables. The case studies are represented by three mathematical models replicating laboratory experiments from the literature. Each model exhibits a different failure mode, and thus a different influence of correlation among material parameters. Section 4 contains an extended discussion of the obtained results reflecting theoretical assumptions of the selected methods and their limitations. Moreover, in the second part of the discussion, we present an artificial analytical example presenting some of these limitations in the case of models with multiple failure modes. In the last section, Section 5, the main findings of this paper are summarized and the importance of the correlation among the material characteristics of concrete is emphasized.

2 | SEMI-PROBABILISTIC APPROACH

Structural reliability represents a crucial topic of civil engineering globally implemented into the design codes using semi-probabilistic approaches. The semi-probabilistic approaches assume the separation of two random variables, structural resistance R and action effect E, through their design values:

$$R_d = F_R^{-1}(-\alpha_R \beta), \tag{1}$$

and

$$E_d = F_E^{-1}(-\alpha_E \beta), \tag{2}$$

where F^{-1} represents the inverse cumulative distribution function, α is a sensitivity factor originally derived from First Order Reliability Method (FORM), and β is the target reliability index. The paper is focused on the estimation of R_d when the function of structural resistance $r(\mathbf{X})$ of input random vector ($\mathbf{X}$ being a vector of N basic variables) is solved by an NLFEA. The recommended value of $\alpha_R = 0.8$ is then utilized, typically with a lognormal distribution of R. Based on these assumptions, the probability distribution is fully described by the mean value and CoV, and the reliability analysis reduces to the estimation of the first two statistical moments—the task of the ECoV methods.

2.1 | Partial safety factors

Although NLFEA has been employed for the design and assessment of structures more frequently in recent decades, it is still insufficiently included in Eurocodes, and its potential for a wide application in the industry is thus limited. Specifically, there is the Partial Safety Factors (PSF) method, and the global factor method for NLFEA of concrete structures according to EN 1992-2:2005[1] implemented in Eurocode. Unfortunately, both methods may provide only crude estimates in the cases with a strongly non-linear behavior, multiple failure modes, or when the assumptions adopted by these approaches do not apply.

Although the PSF implemented into Eurocodes was originally not intended for NLFEA applications, it is often employed due to its simplicity. To estimate the design value of resistance R_d in Equation (3), only one calculation must be computed with the design values of basic variables The design values of input variables are typically derived from the characteristic values using partial factors $\gamma_M = \gamma_{R_d}\gamma_m$, which reflect the uncertainties in the material and the geometrical properties, and model uncertainties:

$$R_d = r(X_{1,k}/\gamma_M, X_{2,k}/\gamma_M, \ldots) = r\left(f_{ck}/\gamma_C, f_{yk}/\gamma_S, \ldots\right). \tag{3}$$

where X_k denotes a characteristic value of the basic variable X, γ_M is the partial factor for a material property, f_c is the concrete compressive strength, and f_y is the yield strength of steel reinforcement.

Note that the design values used in the PSF method might be very low, which might lead to unrealistic

Parameter	CoV	Bias factor
Yield str. f_y	$v_{f_y} = 0.045$	$f_{y,m}/f_{y,k} = exp\left(1.645 v_{f_y}\right)$
Model unc.[a]	$v_\theta = 0.045$	$\mu_\theta = 1.09$
Effect. depth d[b]	$v_d = 0.05$	$\mu_d = 0.95$
Resistance characteristics	$v_{f_y,R} = \sqrt{0.045^2 + 0.05^2} = 0.067$	$\mu_{f_y,R} = f_{y,m}/f_{y,k} \times \mu_d = 1.02$

Abbreviation: CoV, coefficient of variation.
[a]Model uncertainties are reflected in this study by γ_{R_d} and are determined separately for each numerical example.
[b]Valid for d = 200 mm. For other effective depths: $v_d = 0.05\,(200/d)^{2/3}$ and $\mu_d = 1 - 0.05\left(\frac{200}{d}\right)^{\frac{2}{3}}$.

TABLE 1 Parameters assumed for the derivation of a partial factor for reinforcement

results, since non-linear material models are often calibrated to specific ranges of input values only (closer to their mean and/or characteristic values). Therefore, it is beneficial to calibrate the partial factors with respect to real laboratory experiments involving material and structural measurements.[21]

Another approach for the derivation of PSF with the explicit definition of the model, material, and geometrical uncertainties was recently introduced in Annex A to prEN 1992-1-1:2021.[20] The main idea is to account for the biases and CoVs of various basic variables directly in the model of resistance, not only the material itself. Such an approach leads to a considerable simplification of the modeling of the geometrical uncertainties, which may be a difficult task in NLFEA. Disregarding now the model uncertainties—treated separately later by a model uncertainty factor γ_{Rd} specific to the case under consideration—allows for an unambiguous comparison of all the presented safety formats and semi-probabilistic methods (Section 3). The general formula for the definition of PSF according to the new Eurocode proposal is then:

$$\gamma_c = \frac{exp\left(\alpha_R \beta v_{Rc}\right)}{\mu_{Rc}} = \frac{exp\left(\alpha_R \beta \sqrt{v_{fc}^2 + v_\eta^2 + v_{Ac}^2}\right)}{\frac{f_{cm}}{f_{ck}} \mu_\eta \mu_{Ac}} \quad (4)$$

$$\gamma_s = \frac{exp\left(\alpha_R \beta v_{Rs}\right)}{\mu_{Rs}} = \frac{exp\left(\alpha_R \beta \sqrt{v_{fy}^2 + v_d^2}\right)}{\frac{f_{ym}}{f_{yk}} \mu_d}$$

where X_m and v_X denote the mean and the coefficient of variation of the basic variable X, R_c and R_s are model resistances related to concrete crushing and yielding of reinforcement respectively, and μ in Equation (4) is a bias in the basic variable X—the systematic deviation of random values of the variable from its characteristic (nominal) value, expressed as the ratio of the mean to the characteristic (nominal) value.

Equation (4) assumes that concrete resistance R_c, typically governing the resistance of non-slender columns, and reinforcement resistance R_s, typically governing the flexural resistance, are random variables that are lognormally distributed, obtained as a linear product of the relevant resistance parameters; see Tables 1 and 2 taken from Annex A to prEN 1992-1-1:2021.[20]

In the case of a bending failure governed by reinforcement, geometrical uncertainties relate to the most important parameter—the effective depth d as described in Table 1, along with a relevant material property and model uncertainties. In the case of compressive failure with dominating concrete strength, the CoV of resistance is similarly affected by a geometrical uncertainty through the concrete area A_c, but the uncertainty of in-situ strength is also additionally affected by the conversion factor η (see Table 2).

Non-linear material models of concrete typically consider additional material characteristics, such as tensile strength f_{ct}, and fracture energy G_f of concrete. An identical philosophy as in the case of compressive strength was adopted in order to derive the statistics for the PSF and ECoV methods. Specifically, the values according to Table 3 are taken into account.

The CoV of f_{ct} is set to $v = 0.18$ in compliance with prEN 1992-1-1.[20] Note that the variability of G_f is assumed to be identical as for f_{ct}. The characteristic value of tensile strength is obtained from compressive strength according to prEN 1992-1-1[20] as [in MPa]:

$$f_{ct,k} = 0.7\left(0.3 f_{c,k}^{\,2/3}\right) \quad (5)$$

and fracture energy according to the 2021 draft of Model Code 2020 as follows [in MPa]:

$$G_{f,k} = 85 f_{c,k}^{\,0.15} \quad (6)$$

Note that values from Tables 1–3 are further utilized for derivation of mean and characteristic values used in advanced semi-probabilistic methods as described in the next subsection.

TABLE 2 Parameters assumed for the derivation of a partial factor for concrete

Parameter	CoV	Bias factor
Compr. str. f_c	$v_{f_c} = 0.1$	$f_{c,m}/f_{c,k} = exp\left(1.645 v_{f_c}\right)$
Conversion fact. η	$v_\eta = 0.12$	$\mu_\eta = 0.95$
Conc. area A_c	$v_{A_c} = 0.04$	$\mu_{A_c} = 1.0$
Model unc.[a]	$v_\theta = 0.06$	$\mu_\theta = 1.02$
Resistance characteristics	$v_{f_c,R} = \sqrt{0.1^2 + 0.12^2 + 0.04^2} = 0.16$	$\mu_{f_c,R} = 1.18 \times 0.95 \times 1.0 = 1.12$

Abbreviation: CoV, coefficient of variation.
[a]Model uncertainties are reflected by γ_{R_d} determined separately for each numerical example.

TABLE 3 Additional parameters assumed for the derivation of a partial factor for concrete

Parameter	CoV	Bias factor
f_{ct}, G_f	$v = 0.18$	$f_m/f_k = exp(1.645 v)$
Conversion fact. η	$v_\eta = 0.12$	$\mu_\eta = 0.95$
Conc. area A_c	$v_{A_c} = 0.04$	$\mu_{A_c} = 1.0$
Model unc.[a]	$v_\theta = 0.06$	$\mu_\theta = 1.02$
Resistance characteristics	$v_{X,R} = \sqrt{0.18^2 + 0.12^2 + 0.04^2} = 0.22$	$\mu_{X,R} = 1.34 \times 0.9 \times 1.0 = 1.28$

Abbreviation: CoV, coefficient of variation.
[a]Model uncertainties are reflected by γ_{R_d} determined separately for each numerical example.

2.2 | Simplified methods for estimation of coefficient of variation

Besides PSF, there are alternative methods published in scientific papers and international documents such as *fib* Model Code 2010.[22] The ECoV methods under consideration were developed in order to effectively estimate the first two statistical moments of function of random variables from simple formulas. Simplified ECoV methods are often applied in practical design and the assessment of structures without the knowledge of their theoretical background. However, it is essential to respect their limitations to avoid making crude estimates of the design resistance.

2.2.1 | ECoV according to *fib* model code

Probably the most frequently used method is the one developed by Červenka[7] and implemented into *fib* Model Code 2010.[22] It is based on a simplified formula for the estimation of a characteristic value corresponding to a lognormal variable with the mean value μ_R and CoV of the model resistance v_R. Based on two numerical simulations—one with the mean values of the basic variables, R_m, and the other using the characteristic values (5% fractile for material parameters) of basic variables, R_k—the following simplified formula was derived:

$$v_R = \frac{1}{1.645} ln\left(\frac{R_m}{R_k}\right) \tag{7}$$

Based on the conventional models for basic variables provided in Tables 1–3, one can derive the mean and characteristic values summarized in Table 4. Note that the characteristic values X_k^* in Table 4 reflect the uncertainty in the basic variables assumed for the derivation of PSF.

Since there are only two numerical calculations used in Equation (7), it can be shown that ECoV according to fib Model Code 2010 implicitly assumes a full correlation among basic variables (including f_c, f_y or geometrical parameters).[11] Moreover, the simplified Equation (7) for the fractile of a lognormal distribution should be applied for a low CoV only. According to prEN 1990:2021[23] and prEN 1992-1-1:2021,[20] this approximation may be used for a coefficient of variation of less than 0.2; the exact formula for the fractile provided in prEN 1990:2021[23] leads to lower values of v_R.

2.2.2 | ECoV based on Taylor series expansion

The standard method for a statistical analysis of functions of random variables is the Taylor Series Expansion (TSE). The most significant advantages of ECoV based on TSE are its versatility and adaptability. TSE is generally an infinite series which must be truncated to a finite number of terms considering the non-linearity of the

TABLE 4 Input random variables and the defined values for safety formats and ECoV methods

Parameter	Mean value	Characteristic value
Yield strength (Table 1)	$f_{y,m}=\mu_{f_y,R}\times f_{y,k}$	$f^*_{y,k}=f_{y,m}\times exp\left(-1.645\ v_{f_y,R}\right)$
Compressive strength (Table 2)	$f_{c,m}=\mu_{f_c,R}\times f_{c,k}$	$f^*_{c,k}=f_{c,m}\times exp\left(-1.645\ v_{f_c,R}\right)$
Tensile strength (Table 3)	$f_{ct,m}=\mu_{f_{ct},R}\times 0.7\times\left(0.3 f_{c,k}{}^{2/3}\right)$	$f^*_{ct,k}=f_{ct,m}\times exp\left(-1.645\ v_{f_{ct},R}\right)$
Fracture energy (Table 3)	$G_{f,m}=\mu_{G_f,R}\times 85\times f_{c,k}{}^{0.15}$	$f^*_{c,k}=G_{f,m}\times exp\left(-1.645\ v_{G_f,R}\right)$

Abbreviation: ECoV, estimation of coefficient of variation.

approximated function. In engineering applications, it is common to use TSE truncated to linear terms, and thus with $\mu_R\approx R_m$ and CoV:

$$v_R\approx\frac{1}{R_m}\sqrt{\sum_{i=1}^{N}\left(\frac{\partial r(X)}{\partial X_i}\sigma_{X_i}\right)^2},\qquad(8)$$

where the partial derivatives can be numerically computed by various differencing schemes.[13] The simplest scheme is one-sided backward differencing:

$$\frac{\partial r(X)}{\partial X_i}=\frac{R_m-R_{X_i\Delta}}{\Delta_{X_i}}.\qquad(9)$$

$R_{X_i\Delta}$ is the result of a numerical simulation using the mean values of all random variables except the value of the i-th random variable, which is reduced by Δ_{X_i}. Naturally, one can derive various differencing schemes adapted for different situations. Specifically, for ECoV based on TSE, the methodology based on linear and quadratic TSE was recently proposed, providing for three levels of complexity and accuracy.[13] The balance between efficiency and accuracy is achieved by the second level based on linear TSE and the following advanced differencing scheme:

$$\frac{\partial r(X)}{\partial X_i}=\frac{3R_m-4R_{X_{i\frac{\Delta}{2}}}+R_{X_i\Delta}}{\Delta_{X_i}},\qquad(10)$$

where the middle term $R_{X_{i\frac{\Delta}{2}}}$ is obtained by evaluating the mathematical model with a reduced i-th variable $X_{i\frac{\Delta}{2}}=\mu_{X_i}-\Delta_{X_i}/2$ and with the mean values of all the other variables.

The adaptivity of TSE is enhanced by introducing a step size parameter c used for defining the difference $\Delta_{X_i}=\mu_{X_i}-X_{i\Delta}$, where $X_{i\Delta}=F_i^{-1}(\Phi(-c))$. F_i^{-1} is an inverse cumulative distribution function of the i-th variable and Φ is the cumulative distribution function of the standardized normal distribution. Schlune et al.[12] proposed to consider $=(\alpha_R\beta)/\sqrt{2}$. Occasionally, it brings additional computational burden when analyzing different limit states with different β, since it is necessary to calculate $N+1$ (Equation (9)) or $2\ N+1$ (Equation (10)) simulations for each limit state. It might be recommended to use $c=1.645$ irrespective of the type of the investigated limit state, which is in accordance with the ECoV according to *fib* Model Code 2010.

2.2.3 | Eigen ECoV

The recently proposed Eigen ECoV[11] is derived directly from TSE. However, in contrast to TSE suitable for arbitrary correlation structures, Eigen ECoV assumes fully correlated input random variables similarly to ECoV according to *fib* Model Code 2010. Therefore, the number of simulations is significantly reduced in comparison to TSE. The reduction of the number of simulations is achieved by the projection of the differencing scheme into the fully correlated space, i.e. Eigen ECoV is based on the idea of projecting the input random vector on 1D eigen distribution Θ with the variance equal to the first eigenvalue of input covariance matrix $\sigma^2_\Theta=\sum\sigma^2_{X_i}=\lambda_1$, and the mean value is simply obtained as:

$$\mu_\Theta=\sqrt{\sum_{i=1}^{N}\left(\mu_{X_i}\right)^2}.\qquad(11)$$

In the original proposal, there are three levels of Eigen ECoV.[11] The most promising Eigen ECoV formula for the estimation of v_R offering a balance between the efficiency and accuracy (derived directly from Equation (10)) is:

$$v_R\approx\frac{3R_m-4R_{\Theta\frac{\Delta}{2}}+R_{\Theta\Delta}}{\Delta_\Theta}\cdot\frac{\sqrt{\lambda_1}}{R_m},\qquad(12)$$

where the simulation $R_{\Theta\Delta}=r\left(X_{\Theta\Delta}\right)$ with the coordinates of the input realization $X_{\Theta\Delta}=\left(X_{1\Delta},\dots,X_{N\Delta}\right)$ and $R_{\Theta\frac{\Delta}{2}}=r\left(X_{\Theta\frac{\Delta}{2}}\right)$ with the coordinates $X_{\Theta\frac{\Delta}{2}}=\left(X_{1\frac{\Delta}{2}},\dots,X_{N\frac{\Delta}{2}}\right)$ are depicted together with an illustration of the Eigen ECoV method in Figure 1.

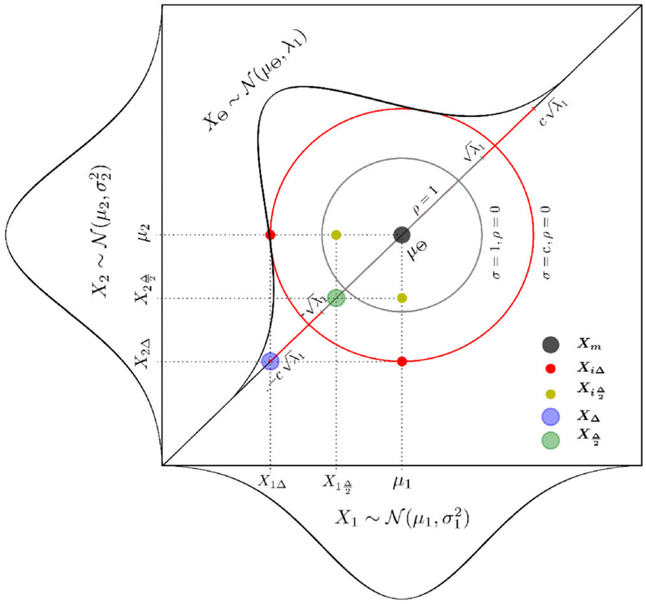

FIGURE 1 Illustration of Eigen ECoV in standardized normal space with coordinates of realizations of input random vector.[11] ECoV, estimation of coefficient of variation

For the sake of clarity, the input vectors consisting of reduced values of input random variables are $X_{i\Delta} = F_i^{-1}(\Phi(-c))$, and the intermediate coordinates are as follows:

$$X_{i\frac{\Delta}{2}} = \mu_{X_i} - \frac{\mu_{X_i} - X_{i\Delta}}{2} = \frac{\mu_{X_i} + X_{i\Delta}}{2}. \tag{13}$$

Δ_Θ represents the distance between μ_Θ and the desired quantile $F_\Theta^{-1}(\Phi(-c))$ obtained as:

$$\Delta_\Theta = \mu_\Theta - \mu_\Theta \cdot exp\left(-c \cdot \frac{\sqrt{\lambda_1}}{\mu_\Theta}\right). \tag{14}$$

The Eigen ECoV combines the versatility and adaptability of TSE through various differencing schemes and the step size parameter c, together with the efficiency of ECoV according to *fib* Model Code 2010. Note that more theoretical details can be found in the original proposal of Eigen ECoV including additional formulas based on other differencing schemes or higher TSE, which is suitable for input variables with high CoV.[11] Similarly, as for ECoV according to Model Code 2010, $c = 1.645$ in numerical examples, which leads to $X_{i\Delta} = X_{i,k}^*$ and $\mu_{X_i} = X_{i,m}$ as summarized later in Table 4.

2.3 | Stratified sampling for estimation of coefficient of variation

The standard approach to the statistical analysis of complex functions of random input variables is the MC simulation consisting of a large number of repetitive deterministic calculations with randomly generated realizations of the input random vector. In order to improve the efficiency of the crude MC method, a stratified sampling (Latin Hypercube Sampling, LHS) was developed.[6] Although the MC simulations lead to an accurate estimation of the statistical moments, it is typically necessary to perform tens to hundreds of simulations, which is often not feasible in combination with NLFEA due to an enormous computational burden. In contrast, LHS is the only general tool for a complex stochastic analysis without any simplifying assumptions (taking arbitrary correlation into account), allowing for estimating statistical characteristics from tens of simulations. This is why it will be used as a reference in the following numerical examples. Note that in order to obtain the consistency of the results and the design values of resistance, we assume lognormal distributions of input random variables with the mean values and CoVs given in Tables 1–3.

3 | CASE STUDIES

The models developed in the ATENA Science software based on non-linear fracture mechanics[24] are used to replicate the experimental results from the scientific literature. The nonlinear behavior of the concrete material is modeled using the fracture-plastic material model.[25,26] Specifically, three typical structural members, each failing in a different mode, are selected. The presented advanced ECoV methods and PSF are compared to a reference LHS solution. Moreover, the specific values of input material characteristics for each of the methods are summarized in tables in order to simplify their practical application or replication of the obtained results.

3.1 | Methodology for numerical comparison of ECoV methods

The task of a probabilistic analysis is simplified to estimating the first two statistical moments, and all the described methods were employed for the comparison of the obtained results in terms of the design resistances determined by Equation (1) and considering $\alpha_R = 0.8$ and $\beta = 3.8$. Model uncertainties are included by an additional reduction factor γ_{R_d} obtained during the extensive benchmark investigation specifically for the employed ATENA Science software and for different failure modes[27] (assuming model uncertainty as a non-dominant resistance parameter). Note that it is recommended to perform Bayesian updating of the prior distribution of the resistance model uncertainty given in the draft Model

TABLE 5 Correlation matrix considered in the case studies[2]

	f_t	f_c	G_f
f_t	1	0.7	0.8
f_c	*0.7*	*1*	0.6
G_f	0.8	0.6	1

Code 2020 as recently described by Engen et al.[28]; the application of this updating is beyond the scope of this paper.

For each example, three reference solutions provided by LHS were obtained reflecting the assumed correlation structure among basic variables. The first case corresponds to the unit covariance matrix, i.e. the case with independent basic variables, which is rather unrealistic for concrete structures (though often assumed). The second limit case represents the full correlation among all input random variables. The full correlation does not represent real situations in physical systems, but it allows for considerable simplifications[11] and it could be considered as another reference solution for the ECoV methods. Both limit cases define the variance interval within which the design values can range depending on the actual correlation structure. The last case solved by LHS represents a realistic correlation matrix inferred from experiments.[2] However, this information is commonly unavailable, and thus not reflected in industrial applications. The realistic correlation matrix according to Slowik et al.[2] prescribes high positive Spearman correlation coefficients only among concrete parameters as can be seen in Table 5.

To sum up, the three cases solved by LHS represent the reference solutions dependent on the correlation among input random variables, i.e. addressing none, full, and realistic correlation. It was recently shown that the simplified ECoV methods implicitly assume full correlation.[11] In contrast to this method, the only method designed for uncorrelated (and possibly arbitrary correlated) variables is TSE, and thus its estimation should be close to the second boundary of the defined correlation interval. Note that TSE with advanced differencing determined almost identical design values as TSE with simple differencing in the following examples, and thus the results of the former are not presented.

In the following figures presenting the obtained results, the reader can find load-deflection diagrams of reference solutions consisting of 30 simulations generated by LHS. In order to clearly show the influence of correlation, three selected realizations are highlighted: the first simulation, 15th (median), and the last realization of the input random vector, where realizations are in an increasing order of the compressive strength of concrete.

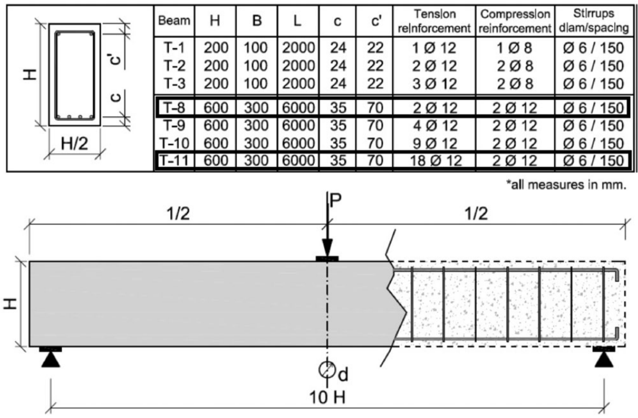

FIGURE 2 Geometry of the beam specimens by Bosco and Debenardi[29]

Besides the load-deflection diagrams and the corresponding statistical values, one can see a comparison of the design values estimated by simplified methods and the defined correlation interval of design values (reference solution). Note that if the estimated design value is out of the interval, it is highlighted by green or red color indicating whether it is conservative or non-conservative, respectively.

For the sake of clarity, Table 4 summarizes the general formulas for the determination of the mean values and characteristic values f_k^* of all the basic random variables used for the ECoV methods in the following examples. Note that the characteristic values with the superscript * are obtained as 5% quantiles of lognormal distributions based on the conventional models adopted for PSF (Section 2.1). Note that in all computations presented here, the nominal values of geometrical variables are applied in NLFEA models. The influence of their bias and CoV, μ_{geo} and ν_{geo}, is already reflected in the characteristic values f_k^*.

3.2 | Experimental program by Bosco and Debenardi

The first two examples are a replication of the tests done by Bosco and Debenardi.[29] The investigated structural member is a simple beam failing in bending. The geometry and reinforcement arrangement of the analyzed beams are described in Figure 2. Two tests with identical beam geometry were selected for this comparison: the T8 test with a low reinforcement ratio, which exhibits a bending failure due to reinforcement yielding, and the T11 test with the reinforcement ratio exhibiting a bending failure due to concrete crushing (see Figure 3). The reinforcement is modeled using the embedded approach assuming a perfect connection to the surrounding

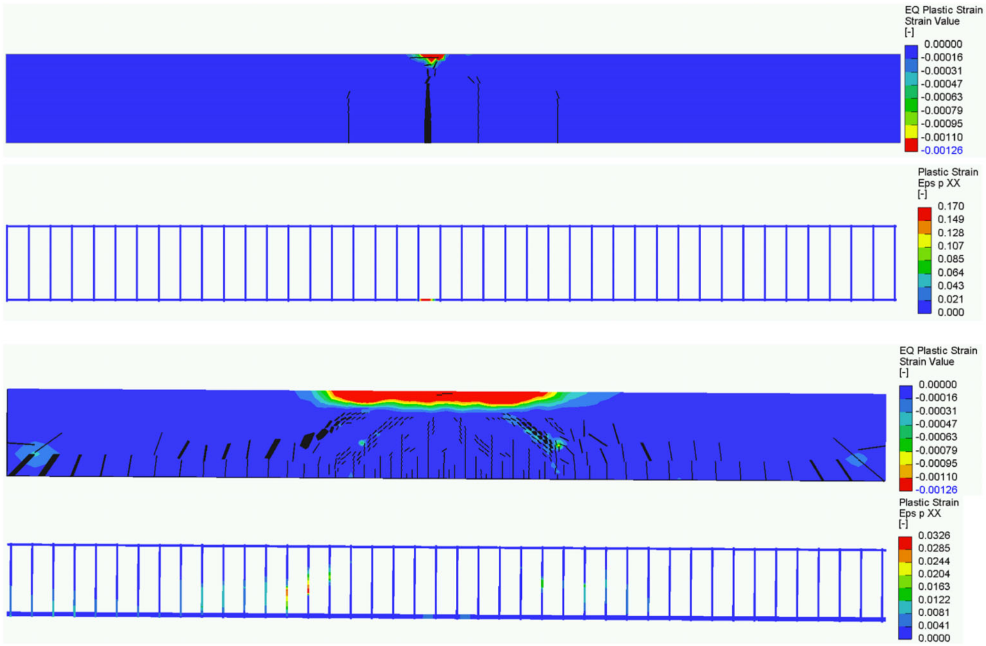

FIGURE 3 Finite element model and failure modes of theT8 beam (top) with reinforcement failure, and T11 (bottom) with concrete crushing failure

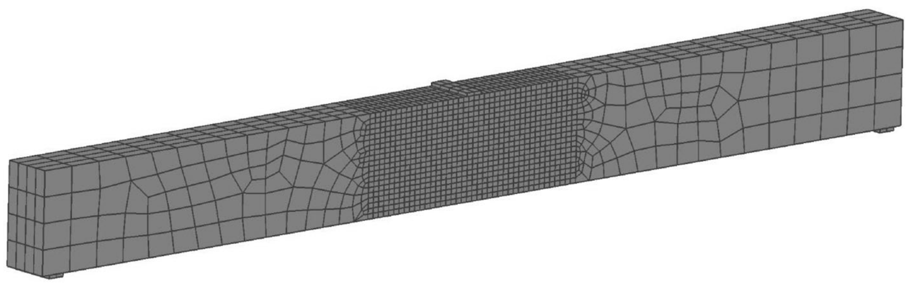

FIGURE 4 Typical mesh for the beam specimens with the mesh size of 30 mm in the middle

concrete. It should be noted that this assumption is not limiting since the bond failure can be captured quite well by the cracking of the finite elements next to the elements with the embedded reinforcement. The typical finite element mesh, shown in Figure 4, leads to an optimal number of finite elements (affecting computational costs) assuring the desired accuracy in the area of interest where the cracks develop.

The characteristic values considered to determine the mean values of strengths are $f_{y,k} = 500\,\text{MPa}$ and $f_{c,k} = 25\,\text{MPa}$. Note that the effective depth of the T8 and T11 beams is $d = 0.565\,\text{m}$, and thus $\nu_d = 0.05\,(200/d)^{2/3} = 0.025$ and $\mu_d = 1 - 0.05\,(200/d)^{2/3} = 0.975$. The important quantiles for the application of the presented safety formats and semi-probabilistic methods for the first two numerical examples are summarized in Table 6, and the results of NLFEA can be found in Table 7.

3.2.1 | The T8-A1 beam failing in bending

The obtained design values are further divided by $\gamma_{R_d} = 1.01$ reflecting the model uncertainties in bending.[27] In this simple example, all the utilized semi-probabilistic methods lead to an identical design value of resistance, $R_d = 40\,\text{kN}$. The experimental result from the original publication was 50 kN.

As can be seen from the reference solutions in Figure 5 (top), there is a significant influence of correlation among input random variables on the variance of the quantity of interest (QoI)—the ultimate resistance of the structural member represented by the peak of the Load-Deflection diagram (LD). The highest variance is associated with the case of full correlation among all input random variables. One can see that the QoI of the highlighted LD realizations clearly corresponds to their rank since the correlation assures that all material characteristics increase proportionally. The second extreme case is the assumption of uncorrelated input random variables, which leads to the lowest variance of QoI, and the rank of realizations is not related to their ultimate resistance. The realistic correlation matrix leads to the variance close to the uncorrelated case due to a low influence of the concrete material characteristics on failure of this structural member, and thus it can be expected that ECoV methods and PSF will be conservative in comparison to this realistic design resistance.

Random variable	$X_{i,d}$	$X_{i,m}$	$X_{i\Delta} = X_{ik}$	$X_{i\frac{\Delta}{2}}$
Yield strength, f_y [MPa]	449	525	482	504
Compressive strength, f_c [MPa]	*17.2*	*28*	21.5	24.7
Fracture energy, G_f [MN/m]	9.02 e-5	1.76 e-4	1.23 e-4	1.49 e-4
Tensile strength, f_{ct} [MPa]	1.2	2.3	1.6	1.9

TABLE 6 Input random variables and the defined values for safety formats and ECoV methods

Abbreviation: ECoV, estimation of coefficient of variation.

TABLE 7 Results of NLFEA utilized in the presented safety formats and ECoV methods

		ECoV MC 2010		Eigen ECoV		
	PSF	R_m	R_k	R_m	$R_{\Theta\frac{\Delta}{2}}$	$R_{\Theta\Delta}$
T8 beam [kN]	40.0	50.1	45.4	50.1	47.8	45.4
T11 beam [kN]	*308*	*376*	*340*	*376*	358	*340*

Abbreviations: ECoV, estimation of coefficient of variation; NLFEA, non-linear finite element analysis.

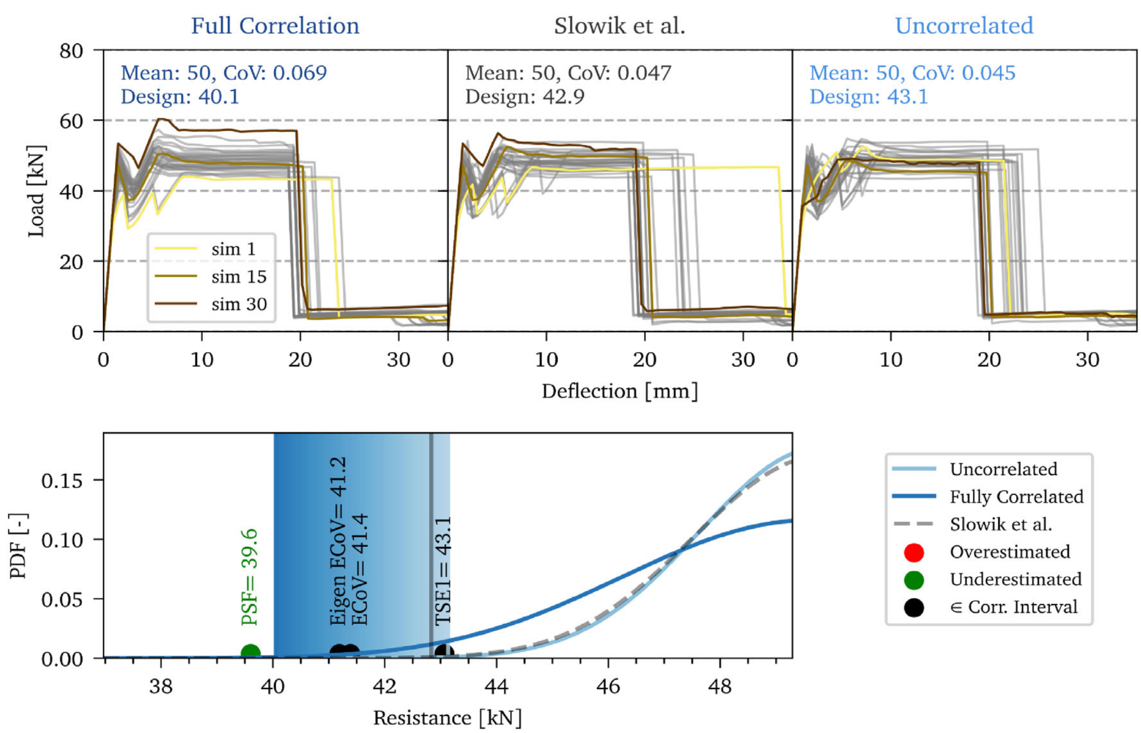

FIGURE 5 T8 beam—Three reference solutions obtained by LHS assuming different correlations (top) and comparison of design resistances (bottom). LHS, Latin Hypercube Sampling

The comparison of design values determined by ECoV methods and PSF can be seen in Figure 5 (bottom). The non-linearity of this example (steel failure in bending) is insignificant, and thus all the employed methods lead to similar design values, and the absolute differences are less than 10%. The highest design resistance is determined by TSE, which is almost identical to the reference uncorrelated solution. With respect to the solution assuming a realistic correlation among input random variables (the vertical gray line in Figure 5), TSE is the most accurate, but also the most expensive method. The ECoV methods estimated almost identical design values inside of the correlation interval. Finally, the most conservative method is PSF, though the results of all the used methods are in close agreement. This example shows the typical results of semi-probabilistic methods in simple, almost linear mathematical models.

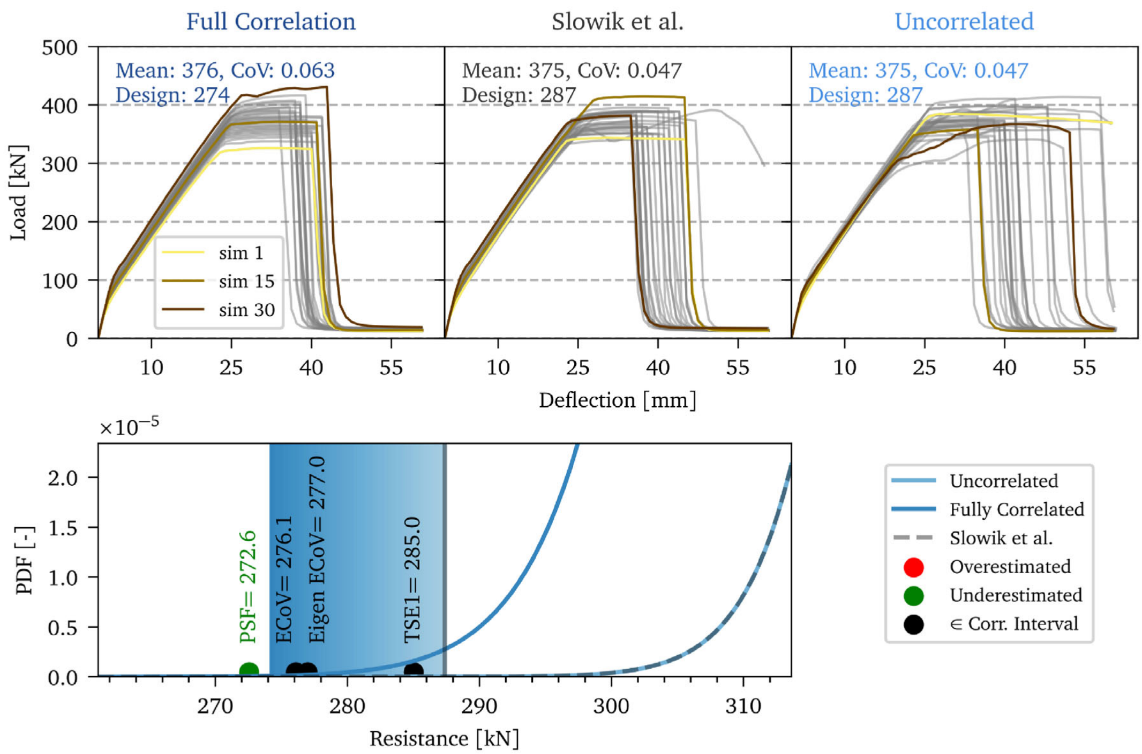

FIGURE 6 T11 beam—Three reference solutions obtained by LHS assuming different correlations (top) and comparison of design values (bottom). LHS, Latin Hypercube Sampling

3.2.2 | The T11-A1 beam failing in bending

The T11-A1 beam exhibits combined compressive crushing and shear failure by the yielding of stirrups. Therefore, the obtained design values are further reduced by $\gamma_{R_d} = 1.13$ reflecting the model uncertainties of shear failure.[27] Design resistances are depicted in Figure 6 together with the reference solution (the distribution and design quantile obtained by LHS). The experimental result from the original publication was 380 kN.

In this example, the yield strength of the reinforcement has the dominant influence on the ultimate structural resistance, since the realistic correlation matrix defining a strong correlation among concrete material characteristics leads to the identical variance as in an uncorrelated case. The comparison of the design values determined by ECoV methods and PSF can be seen in Figure 6 (bottom). Note that all the methods are in good agreement with the LHS and their results are according to expectations: ECoV methods lead to design values near the fully correlated boundary and TSE1 leads to the uncorrelated boundary of the correlation interval. The most conservative design value is obtained by PSF from a single simulation, though it is very close to the defined reference interval, and thus it may be seen as a very efficient method.

3.3 | Experimental program by Anderson and Ramirez

The third example is based on the experimental program by Anderson and Ramirez.[30] In this experiment, a beam with the cross-section of 406 × 406 mm was subjected to a four-point bending test with a shear span a = 0.91 m. The beam was designed to fail in shear, i.e. to comply with the condition for shear stress $V_{\text{test}}/(b_w d) > 6$. The beam geometry and reinforcement are shown in Figure 7 and the finite element model in Figure 8. The shear reinforcement is composed of double stirrups no. 3 with ∅ 9.525 mm. The top longitudinal reinforcement consists of 5 bars no. 6 (∅ 19.05 mm) and the bottom reinforcement of 5 bars no. 9 (∅ 28.65 mm).

The characteristic values used for the determination of the mean values of material parameters are $f_{y,k} = 422\,\text{MPa}$ and $f_{c,k} = 25\,\text{MPa}$. The effective depth of this beam is d = 0.344 m, and thus $\nu_d = 0.05\,(200/d)^{2/3} = 0.034$ and $\mu_d = 1 - 0.05\,(200/d)^{2/3} = 0.97$. The important quantiles for the application of the presented semi-probabilistic methods are summarized in Table 8.

From the obtained results of NLFEA, the beam is failing in shear (see Figure 9), and thus the obtained design values are further reduced by $\gamma_{R_d} = 1.13$ reflecting the model uncertainties of the shear failure.[27] Numerical results of

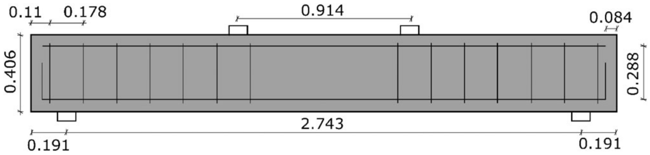

FIGURE 7 Geometry of the W1 beam of Anderson and Ramirez.[30] Note that the dimensions are in [m], but the original experiment was in imperial units

NLFEA for the ECoV methods and PSF are summarized in Table 9. The experimental result was 460 kN.

The results of the semi-probabilistic methods are presented in Figure 10 (bottom). From the reference solutions by LHS, one can see that there is a moderate influence of correlation among concrete material characteristics since the case with the realistic correlation matrix leads to a slightly higher variance in comparison to the uncorrelated case. The design values determined by the simplified ECoV methods are in good agreement with the reference solution. The estimate of the standard ECoV is close to the uncorrelated boundary while it should be closer to the fully correlated reference solution; the difference, however, is insignificant for practical applications. This can be attributed to a significant non-linearity in this example. Note that in contrast to the standard ECoV, an almost identical estimate by TSE1 is in agreement with the theoretical expectations and the high accuracy of this method is demonstrated by all the case studies. The additional simulation in Eigen ECoV significantly improves the estimation of standard ECoV, and its result is close to its reference solution (fully correlated). This is in agreement with the previous theoretical results,[11] since it should achieve a higher accuracy in comparison to the standard ECoV in the case of moderate non-linearity of the investigated mathematical models. Note that PSF leads to a very accurate estimation of design resistance, though it is not typical in the case of shear failure.[2–4]

4 | DISCUSSION

4.1 | Effect of correlation

Statistical correlation among material characteristics might play a crucial role, since it has a significant influence on the variance of QoI, particularly for concrete structures. Nonetheless, the exact information about the correlation matrix is usually unavailable and the recommendations in scientific literature widely differ depending on the concrete mixture, strength class, etc.[2,31,32] For practical analyses of concrete structures, two extremes may be important: fully correlated random variables and uncorrelated random variables, which together define the variance interval caused by insufficient information about the correlation. The fully correlated random input variables usually lead to a larger variance of QoI and conservative estimates of design values. The lower boundary of the variance corresponding to the uncorrelated case needs to be estimated by advanced probabilistic methods such as LHS, or approximated by TSE. In both cases, the number of calculations is significantly higher in comparison to the ECoV methods.

Based on the obtained results for the three structural members failing in different modes, it can be concluded that all the presented ECoV methods are well bounded by the correlation intervals. One should keep in mind that, Eigen ECoV and ECoV according to *fib* Model Code 2010 are based on the fully correlated case, and TSE correspond to the uncorrelated case. The examples indicate that Eigen ECoV provides better estimates for the fully correlated case and one additional simulation may significantly improve the estimate by ECoV (according to *fib* Model Code 2010). However, if only a single input random variable fully describes the variance of QoI, the standard ECoV has a superior efficiency as shown by the second example. In contrast, Eigen ECoV might be more suitable for shear failures with a higher non-linearity. The analysis of the obtained results and of the underlying assumptions reveals that the accuracy of the ECoV methods depends on a specific failure mode and assumed correlation matrix. Moreover, the ECoV methods may provide crude estimates in the case of multiple failure modes as briefly discussed in the following subsection.

4.2 | Limitation of simplified ECoV methods for multiple failure modes

Simplified safety formats are commonly devised to yield adequate estimates of design resistances in most practically relevant applications, while, inevitably in some cases, overconservative or unsafe approximations might be obtained. Though a detailed study of such errors is beyond the scope of this contribution, a fundamental example is analyzed here to provide the first insights. It is often argued that simplified safety formats may fail in the cases with several local extrema as typically caused by multiple failure modes. To verify this, two columns exposed to compression without eccentricity, acting as a series system, are analyzed (Figure 11).

The example is focused on a simple series system whose resistance R is obtained as a minimum of resistances of two identical RC columns:

$$R/A_c = \min\left(f_{c1} + \rho f_{y1}, f_{c2} + \rho f_{y2}\right) \tag{15}$$

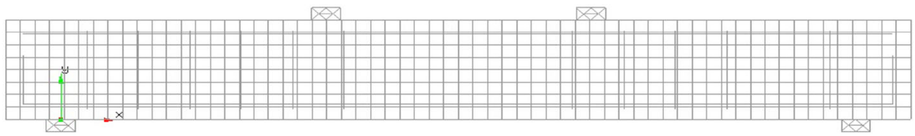

FIGURE 8 Finite element mesh for the nonlinear analysis of the W1 beam of Anderson and Ramirez

TABLE 8 Input random variables and the defined values for safety formats and ECoV methods

Random variable	$X_{i,d}$	$X_{i,m}$	$X_{i\Delta} = X_{ik}$	$X_{i\frac{\Delta}{2}}$
Yield strength, f_y [MPa]	369	439	399	419
Compressive strength, f_c [MPa]	*17.1*	*28*	21.5	24.7
Fracture energy, G_f [MN/m]	9.02 e-5	1.76 e-4	1.23 e-4	1.49 e-4
Tensile strength, f_{ct} [MPa]	1.2	2.3	1.6	1.9

Abbreviation: ECoV, estimation of coefficient of variation.

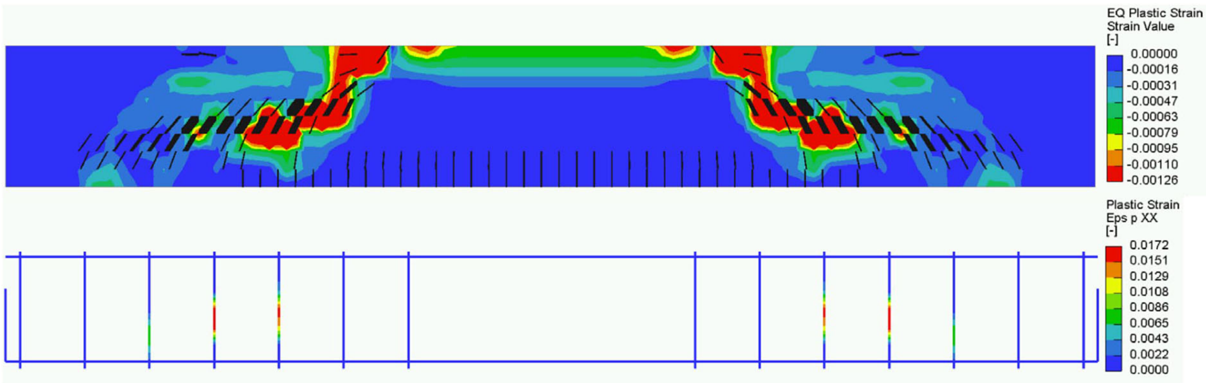

FIGURE 9 Results from the nonlinear analysis of theW1 beam failing in shear

TABLE 9 Results of NLFEA utilized in the presented safety formats and ECoV methods

		ECOV MC 2010		Eigen ECoV		
	PSF	R_m	R_k	R_m	$R_{\Theta\frac{\Delta}{2}}$	$R_{\Theta\Delta}$
W1 beam	307	387	351	387	367	351

Abbreviations: ECoV, estimation of coefficient of variation; NLFEA, non-linear finite element analysis.

where A_c denotes a concrete section area; and ρ is the reinforcement ratio common to both columns. The reinforcement ratio is a study parameter, arbitrarily varied disregarding practical constraints.

Mutually statistically independent strengths f_{ci} and f_{yi} are described by lognormal distributions with the following characteristics:

- $f_{cm} = 29.1$ MPa, $v_{fc} = 15\%$, and $f_{c0.05} = f_{ck} = 22.6$ MPa
- $f_{ym} = 455$ MPa, $v_{fy} = 5.8\%$, and $f_{y0.05} = f_{yk} = 414$ MPa

These assumptions are based on a more detailed study focused on the performance of safety formats for series systems.[33] Uncertainty in geometrical variables is ignored here to keep focus on the key aspects affecting the performance of the simplified safety formats.

All the obtained design values are normalized to those obtained by the probabilistic approach using the numerical integration ($R_{d,prob}$). Besides $R_{d,PSF}$, all design values are obtained as a fractile of the system resistance corresponding to the probability of 1.12‰, resulting from $\alpha_R = 0.8$ and $\beta = 3.8$.Model uncertainty is not considered in this section as it is typically treated separately, beyond the application of a particular safety format. Note that the justification of $\gamma_C = 1.5$ and $\gamma_S = 1.15$ according to Eurocode 2 Commentary[34] indicates that the model uncertainty factors related to the recommended values in EN 1992–2:2005[1] are very close to unity, and thus $\gamma_C = 1.5$ and $\gamma_S = 1.15$ are adopted without any adjustment in the following analysis, where model uncertainty is ignored.

Figure 12 displays a variability of $R_{d,safety\ format}/R_{d,probab}$ with a reinforcement ratio. For low ρ-values, the resistance of a column is governed by the concrete contribution, while the reinforcement contribution becomes important with the increasing ρ, and the distribution of column resistance attains a bimodal character. Figure 12

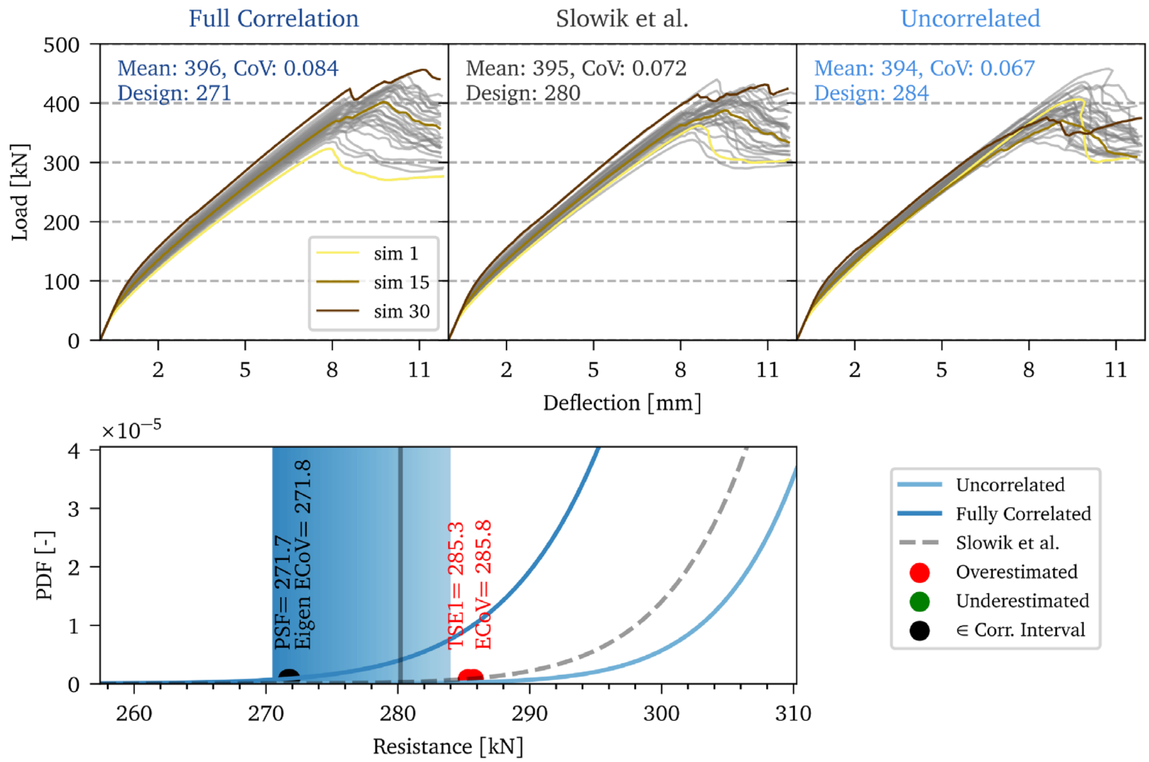

FIGURE 10 W1 beam—Three reference solutions obtained by LHS assuming different correlations (top) and comparison of design resistances (bottom). LHS, Latin Hypercube Sampling

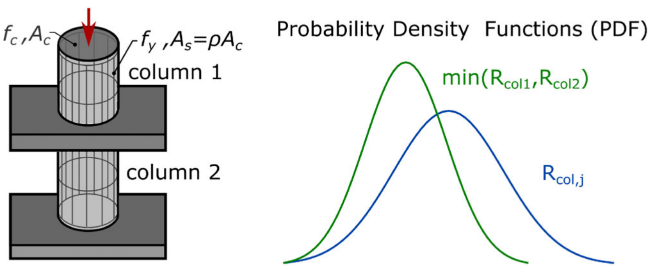

FIGURE 11 Illustration of the analyzed series system and probability density functions (PDFs) of component and system resistances

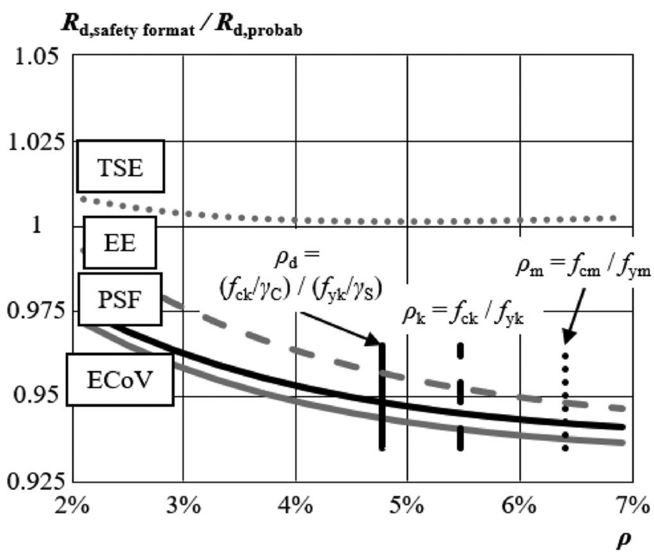

FIGURE 12 Variability of $R_{\mathrm{d,safety\ format}}/R_{\mathrm{d,probab}}$ with ρ

also shows that ECoV based on TSE leads to nearly the same design resistance as the probabilistic approach. All the other simplified formats provide reasonably conservative estimates, with errors mostly between 2% and 6%. Eigen ECoV (EE in the figure) performs slightly better than PSF and ECoV, but the differences are negligible in this case.

All the ECoV methods lead to conservative estimates of ν_R as can be seen in Figure 13. As expected, the best estimates are obtained by TSE based on the largest number of limit state function evaluations, then following with Eigen ECoV and the standard ECoV.

What is interesting to observe is that TSE yields $R_{\mathrm{d,TSE}} \approx R_{\mathrm{d,probab}}$ for any $\rho > \rho_{\mathrm{m}}$ while it systematically overestimates ν_R (which should have led to $R_{\mathrm{d,TSE}} < R_{\mathrm{d,probab}}$). A detailed analysis indicates that this safe-sided error is nearly exactly outweighed by the failure in identifying the type of distribution of system resistance, ignoring the bimodal character of the distribution by TSE; this is common to all ECoV methods. The skewness of the bimodal distribution (reflected by the probabilistic

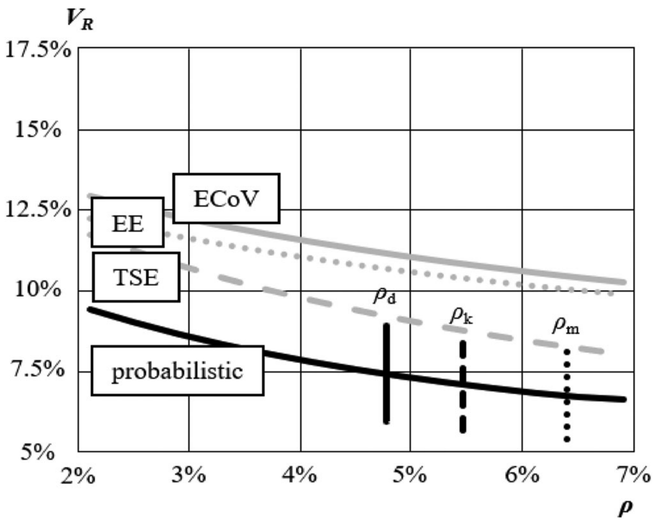

FIGURE 13 Variability of coefficients of variation estimated by various methods with ρ

approach) tends to be lower than that of a lognormal distribution, and the ECoV methods thus make unsafe errors here.

The presented limited analysis of the series system with two failure modes indicates a number of directions for further research:

- Most concrete structural systems are deemed to have properties closer to parallel systems as they are often indeterminate, providing for multiple load paths. The preliminary results for parallel systems, already partly presented,[35] indicate that the ECoV methods perform in a similar way as those observed here; the partial factor method tends to be conservative for parallel systems.
- Positive correlations between failure modes are expected to reduce the ECoV error for both types of systems. In contrast, the errors may amplify with an increasing number of failure modes of a similar importance. These counteracting effects should be investigated further.

5 | CONCLUSIONS

The comparison of the selected advanced semi-probabilistic methods is presented in three numerical examples failing in different modes. The case studies demonstrate how uncertainties in geometry can be combined with those in material properties and considered in NLFEA applications. The obtained results show that all the employed methods lead to design values close to the reference solution. The numerical differences become more significant with an increasing non-linearity of the failure mode. The theoretical behavior of the recently proposed modification of Taylor Series Expansion (TSE) and its adaptation Eigen ECoV is successfully verified by realistic case studies. The correlation among input random variables might play a crucial role in determining the design values, and thus it might be beneficial to verify two limit situations: a fully correlated case by ECoV methods, and an uncorrelated case by TSE. For practical applications, recommendations should be provided as to when the examination of the two limiting situations is needed and how to proceed when a large difference between the design values is obtained.

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DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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