Vladimir Cervenka

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Reliability-based non-linear analysis according to *fib* Model Code 2010

The fib Model Code 2010 for Concrete Structures introduces numerical simulation as a new tool for designing reinforced concrete structures. The model of resistance based on non-linear analysis requires adequate model validation and a global safety format for verifying designs. The numerical simulations combined with random sampling offer the chance of an advanced safety assessment. Approximate methods of global safety assessment are discussed and compared in a case study. An example of a bridge design supported by non-linear analysis is shown.

Keywords: non-linear analysis, safety formats, reliability, *fib* Model Code 2010

1 Introduction

Advanced non-linear analysis is becoming a useful tool for the design of new and assessment of existing structures. This development is influenced by the general impact of information technologies on society and the economy. The fast-developing industry of concrete structures yields new structural solutions, which are often verified by numerical simulations based on non-linear analysis and the finite element method. Examples of such applications can be observed in integral bridge structures [1] and nuclear power plants [2]. This trend is confirmed by several recent conferences devoted to computational mechanics, such as Euro-C 2010 and WCCM 2012, where special sections were devoted to concrete structures. The subject was recently dealt with in *fib* Task Group 4.4 [3].

Non-linear finite element analysis can be used in the design of concrete structures as an alternative to linear analysis. The concept has been developed within the field of computational mechanics with the aim of simulating real structural behaviour. Although it was initially used in research studies to support experimental investigations and explain observed structural behaviour, it has recently become a powerful design tool.

In the design process, non-linear analysis offers the engineer a refined verification of a structural solution by simulating structural response under design actions. Such a simulation can be regarded as a virtual test and does not fit into the traditional scope of the design process. This is mainly due to the basic differences between linear and non-linear approaches. In traditional design, distribution of internal forces is carried out by linear analysis and safety is checked locally in sections. There are two important discrepancies worth mentioning in this approach. First, the elastic force distribution is one of the many possible states of equilibrium, which can be realistic at low load levels only. A significant force redistribution can occur due to inelastic response. Second, the local section safety check of limit states is made under the assumption of non-linear material behaviour (cracking, reinforcement yielding, etc.), which is not consistent with the elastic analysis of internal forces. Furthermore, the local safety check does not provide any information about overall structural safety. Nevertheless, this approach represents a very robust design method verified through many years of experience, and is the basis of the partial factor design concept currently in use.

In order to support a more rational safety assessment, *fib* Model Code 2010 [4] reflects new developments in safety formats based on probabilistic methods. Chapter 4 "Principles of structural design" introduces the probabilistic safety format as a general and rational basis for evaluating safety. In addition to the partial factor format, which remains as the main safety format for most practical cases, a "global resistance format" is recommended for non-linear analysis. Section 7.11 "Verification assisted by numerical simulations" outlines a guide for using non-linear analysis for assessing resistance. This paper illustrates the background to these innovative approaches.

2 Numerical simulation

The finite element method is typically used for the numerical solutions to continuum problems. Depending on the type of formulation (stiffness, compliance and mixed methods), the results are, by definition, different from the exact solution. In the stiffness formulation a best possible equilibrium is found for a given approximation (finite element type and size). The finite element solution should satisfy the requirement of convergence to the exact solution by reducing the element size (and increasing the number of degrees of freedom). Thus, irrespective of the material model, the approximations introduced solely by the finite element formulation can be a significant source of errors in numerical analysis and these errors should be

Corresponding author: cervenka@cervenka.cz

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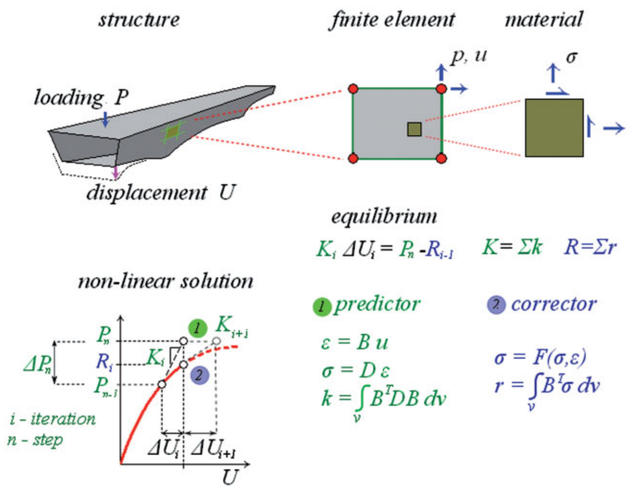


Fig. 1. Typical algorithm for non-linear finite element analysis

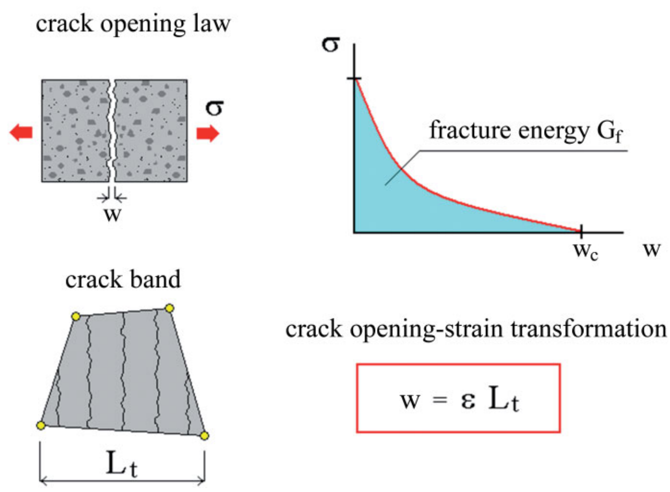


Fig. 2. Model of fracture energy-based crack band

adequately controlled. Non-linear analysis introduces additional effects, e.g. material behaviour approximation, large deformations (change of geometry), or time-dependent behaviour (e.g. creep). The most significant effect in concrete structures is the material behaviour.

The principles of non-linear analysis are illustrated in Fig. 1. The non-linear solution is performed by a predictor-corrector iterative process (variations of the *Newton-Raphson* method). In the predictor (1), the solution is estimated by a linear analysis based, optionally, on tangent or initial material stiffness. The solution is improved in the corrector (2), based on non-linear constitutive laws. The iterative process is stopped when the difference between predictor and corrector is acceptably small. Appropriate iterative techniques can be employed for chosen specific constitutive laws. A balanced approximation of numerical methods involved in all parts of the model, i.e. in structural discretization, element formulation and material laws, should be maintained.

3 Constitutive models

The material models used for concrete, reinforcement and their interaction should capture all significant and relevant features of material behaviour for the problem under consideration. Constitutive laws should be based on the principles of continuum and failure mechanics and must ensure the objectivity of the solution in the context of numerical methods.

Models for material softening, i.e. materials exhibiting a decrease in strength after reaching a certain ultimate stress value, should include appropriate regularization techniques in order to reduce the mesh sensitivity of strain-based formulations of constitutive laws. An example of such a technique is the crack band method used for modelling cracks in concrete as shown in Figs. 2 and 3. A discrete crack is modelled by a band of smeared cracks. Owing to the softening of the stress–crack opening law, the strain localizes in a narrow band of elements but remains evenly distributed within one element. The crack band model ensures that the fracture energy required for crack formation is dissipated within the crack band. This technique significantly reduces the mesh effect [6, 7]. Examples of crack pattern simulations are shown in Fig. 4.

One important property of concrete is its sensitivity to the multi-axial stress state, i.e. a significant strength increase under hydrostatic stress, referred to as the confinement effect, see Figs. 5 and 6. Two well-known models re-

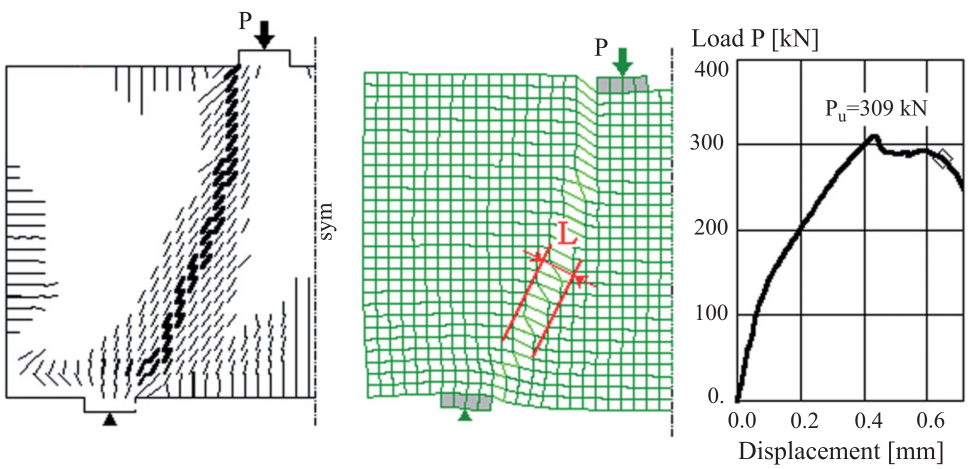


Fig. 3. Example of a crack band in a shear wall [5]

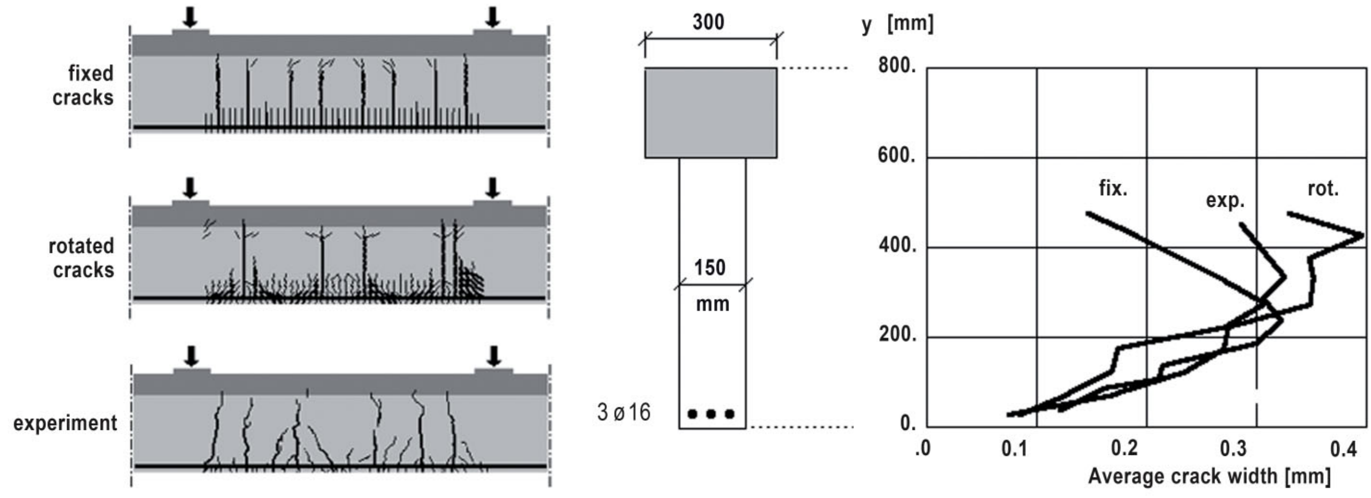


Fig. 4. Crack pattern simulation of beams (test by *Braam* [8])

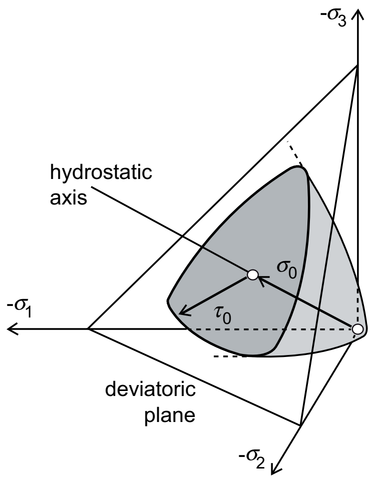


Fig. 5. Concrete failure surface in 3D stress state

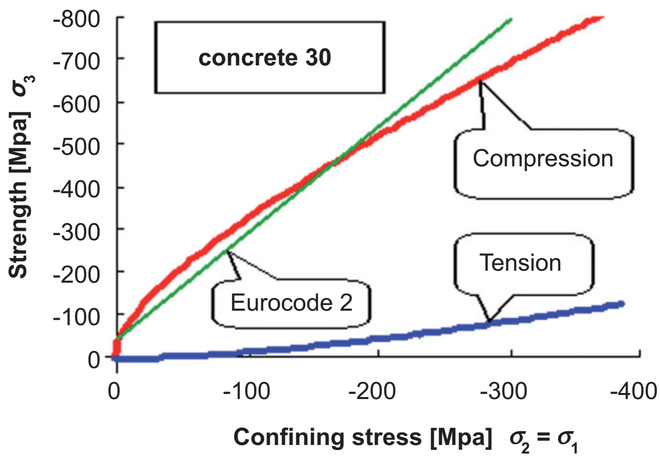


Fig. 6. Example of confinement effect modelled by the *Menetrey-Willam* yield function

flecting this effect are those of *Willam* [9] and *Ottosen* [10]; both supply satisfactory results for a wide range of concrete strengths (including HSC).

In numerical implementations, various effects interact and, in general, can form a complex non-linear problem. Therefore, a strain decomposition method, where the total strain is the sum of strains due to fracture, plasticity, creep, etc., is often used in order to solve this problem. An example of such a constitutive model is the fracture-plastic model proposed in [11].

Only the most significant concrete properties were mentioned in the above discussion. However, there are some additional properties that are important as well, such as modelling of interfaces between two concrete surfaces, steel-concrete contacts, bond between reinforcement and concrete and reinforcement itself. All should be considered in practical applications.

4 Model validation

Numerical models are more complex than simplified engineering methods and the associated uncertainty is potentially high. Therefore, numerical models must be validated to ensure adequate safety. Such a validation should cover the whole range of inherent approximations: constitutive models, numerical discretization and structural solution.

Basic material tests serve to validate the constitutive relations and are performed on simple structures, with the aim of reducing the influence of geometry and boundary conditions under well-defined stress and strain conditions. Examples of such tests are compressive tests on concrete cylinders, fracture tests on concrete prismatic specimens subjected to three-point loading and tension stiffening tests in uniaxial tension for reinforcing bars embedded in concrete members. These tests are typically described in codes for materials testing, such as those recommended by RILEM.

The aim of structural tests is to validate the ability of the algorithm or software to reproduce certain structural behaviour objectively. This is often accomplished by way of benchmark calculations. For example, if a shear wall is to be simulated, then validating the software by means of shear wall experiments should be ensured. Such studies can be considered to be a rational basis for choosing adequate material models and software for a given structure.

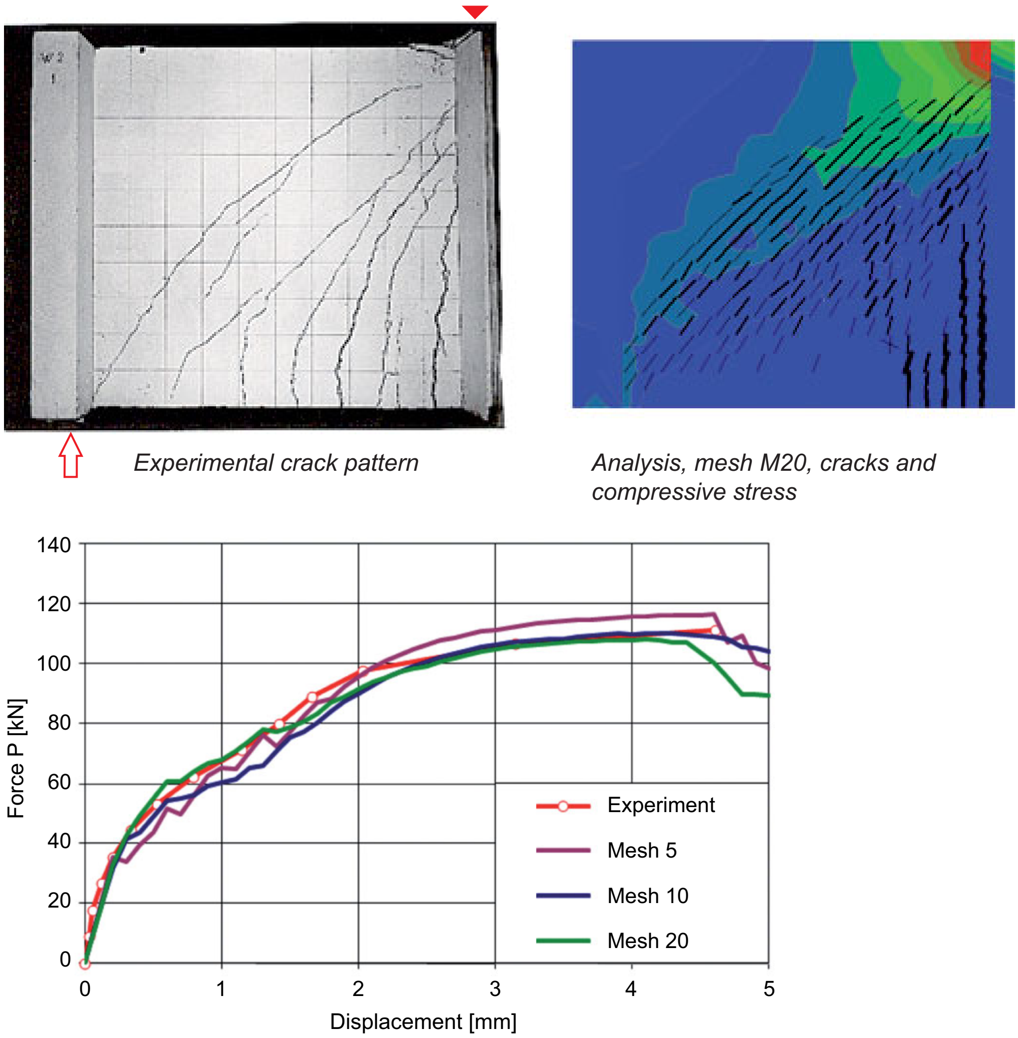


Fig. 7. How mesh size affects of shear panels

Mesh sensitivity tests should be performed in order to validate the finite element mesh of the numerical model. At least three mesh cases with different element sizes should be tested and their effect on the resistance evaluated. In the case of a significant mesh sensitivity, when at least two different mesh sizes do not provide sufficiently similar results, the numerical model should be considered as not objective. An example of a mesh sensitivity study of a shear panel tested by the author is shown in Fig. 7, (for more details see [3], p. 168). It can be seen that the mesh refinement has an opposite effect on resistance (stiffness) in the ranges of crack formation (increased stiffness) and maximum load (reduced strength). Thus, the principles from elastic analysis based on displacement methods, where refining the mesh always reduces the stiffness, cannot be simply extended into a non-linear analysis and should be applied with caution.

The errors of non-linear solutions are controlled by convergence criteria. The solution convergence is satisfied when the error lies within prescribed limits. In the case of the stiffness method, the most significant convergence criterion is the error in the force equilibrium (residual forces). In addition, increments in displacements or the residual energy can be checked. The choice of an adequate error tolerance is an important aspect of non-linear analysis. The admissible errors must be appropriately validated, e.g. by a convergence study in which the results obtained with different tolerances are compared.

Finally, the model performance on the structural level should be checked. It should prove the capability of the chosen numerical model to reproduce the structural behaviour under consideration.

An example of validation based on a shear test from [7] is shown in Fig. 8. The beam size tested by *Collins* and *Yoshida* [12] exceeds the usual beam dimensions (span = 12 m, depth = 2 m). The failure was dominated by brittle response, which contributed to the size effect and which could be well reproduced by the numerical model based on fracture mechanics. More about this study will be show later in the examples of application.

5 Global safety format and model uncertainty

The usual design condition is considered as

$$F_d < R_d \tag{1}$$

where F_d is the design action and R_d is the design resistance and both these entities cover safety margins. In this formulation the safety of loading and resistance are treat-

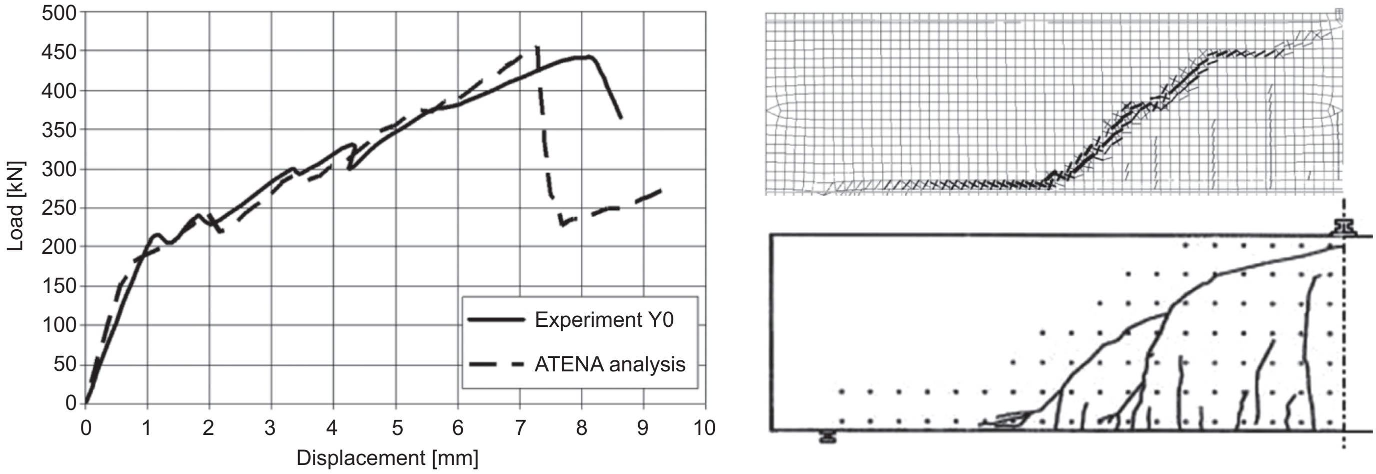


Fig. 8. Comparison of load-displacement diagrams and crack patterns of large beams

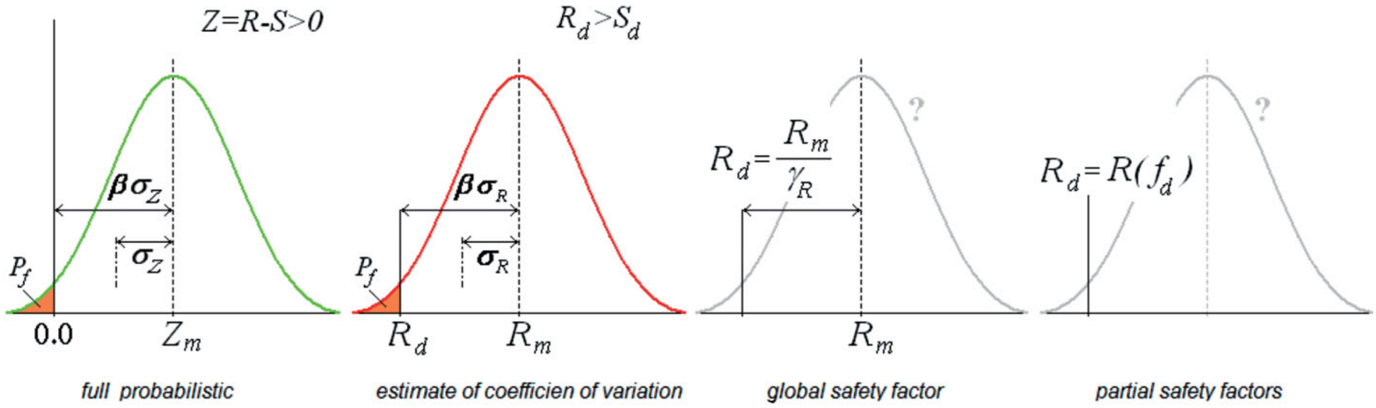


Fig. 9. Safety formats for design resistance

ed separately, which is a certain approximation compared with a general probabilistic approach. In design practice (based on the partial safety factors) we accept this simplification and consider $F_d = F(S,\gamma_G,\gamma_Q,\gamma_P,..)$ where the representative load S is factorized by partial safety factors $\gamma_G,\gamma_Q,\gamma_P,..$ for permanent load, imposed load, prestressing, etc.

In non-linear analysis R_d describes the **global resistance** (e.g. set of forces representing an imposed load, horizontal load, etc.). Note that in the partial safety factor method we assume failure probabilities of separate materials but do not evaluate the failure probability on the structural level. Unlike in sectional design, the global resistance reflects an integral response of the whole structure in which all material points (or cross-sections) interact. The safety margin can be expressed by the safety factor

$$R_d = \frac{R_m}{\gamma_R} \tag{2}$$

where R_m is the mean resistance (sometimes referred to as nominal resistance). The global safety factor γ_R covers all uncertainties and can be related to the coefficient of variation of resistance V_R (assuming a log-normal distribution according to Eurocode 2) as

$$\gamma_R = \exp(\alpha_R \beta V_R) \tag{3}$$

where α_R is the sensitivity factor for resistance and β is the reliability index. It is recognized that variability included in V_R depends on uncertainties due to various sources: material properties, geometry and resistance model. They can be treated as random effects and analysed by probabilistic methods. Owing to the statistical data available, the probabilistic treatment of materials and geometry can be performed in a rational way. However, a random treatment of model uncertainties is more difficult because of limited data. A simplified formulation was proposed in *fib* Model Code 2010, where the denominator on the right-hand side of Eq. (2) is a product of two factors: $\gamma_R = \gamma_m \gamma_{Rd}$ (which follows from the determination of partial safety factors in *fib* Model Code 2010, section 4.5.2.2.3). The first factor γ_m is related to material uncertainty and can be established by a probabilistic analysis. The second factor γ_{Rd} is related to model and geometrical uncertainties and recommended values are in the range 1.05–1.1 (as suggested by Eurocode 2-2).

Recent investigations by *Schlune* et. al. [13] found such values to be unsafe and proposed a more general method in which the overall coefficient of resistance variation can be determined as

$$V_R = \sqrt{V_G^2 + V_m^2 + V_{Rd}^2} \tag{4}$$

where variability due to specific sources are identified: V_G – geometry, V_m – material strength, V_{Rd} – model. This approach allows the inclusion of all uncertainties in a more rational way. Based on a survey of various blind benchmark studies, *Schlune* et al. concluded that the model uncertainties of non-linear analysis are much higher than in standard design based on engineering formulas and are strongly dependent on modes of failure. Reported coefficients of variation due to model uncertainty for bending failure are in the range 5–30 %, and 15–64 % for shear. *Schlune* et al. concluded that due to the lack of data, the choice of the model uncertainty often depends on engineering judgment and can be subjective. However, these conclusions do not recognize the effect of model validation, which can decrease model uncertainties. Further research is needed to recommend appropriate values of the model uncertainty for numerical simulations.

The assessment of the design resistance according to Eq. (1) can be achieved by various methods, ranging from a full probabilistic analysis to the partial factor method, which differ in the level of approximations involved. These safety formats are briefly characterized below and are illustrated in Fig. 9 by comparing how they represent failure probability.

5.1 Full probabilistic analysis

In general, probabilistic analysis is the most rational tool for assessing the safety of structures. It can be further refined by introducing non-linear structural analysis as a limit state function. The numerical simulation resembles real tests on structures by considering a representative group of samples, which can be analysed statistically for assessing safety. An approach applied in [16] will only be briefly outlined here. More information on probabilistic analysis can be found in [17].

The probabilistic analysis of resistance is performed by the LHS method, in which the material input parameters are varied in a systematic way. The resulting array of resistance values is approximated by a distribution function of global resistance and describes the random variation of resistance. Finally, for a required reliability index β, or probability of failure P_f, the value of design resistance R_d should be calculated.

However, full probabilistic analysis is computationally demanding and requires good information about random properties of input variables. It is usually applied in special cases where the consequences of failure justify the effort.

Probabilistic analysis based on numerical simulation with random sampling can be briefly described as follows:

(1) Formulation of a numerical model based on the non-linear finite element method. Such a model describes the resistance function and can perform a deterministic analysis of resistance for a given set of input variables.

(2) Randomization of input variables (material properties, dimensions, boundary conditions, etc.). This can also include some effects that are not in the action function (e.g. prestressing, dead load, etc.). Random material properties are defined by a random distribution type and its parameters (mean, standard deviation, etc.). They describe the uncertainties due to the variation of the resistance properties. The randomization can be carried out by two methods: (1) **random variables**, where the parameter is constant within a sample (structure) but changes between samples; (2) **random fields**, where the parameter is randomly variable within a sample. A correlation of random variables should be considered appropriately.

(3) Probabilistic analysis of resistance. This can be performed by the numerical method of the Monte Carlo-type of sampling, such as the LHS sampling method. The results of this analysis provide random parameters of resistance, e.g. mean, standard deviation, etc., and the type of distribution function for resistance (PDF).

(4) Evaluation of design resistance based on the reliability index β or probability of failure. In this, a design point is found by extrapolating a point around a central region based on the probability distribution function (PDF).

The advantage of a full probabilistic analysis is that it is independent of a failure mode. The potentially higher safety margins of some failure modes, e.g. shear failure, is automatically included in the higher sensitivity of numerical resistance to a brittle failure. A disadvantage of this approach [16] is that the target value of design resistance is located in the tail of the PDF. This function is determined by the best fit from the available, and the design value is obtained by extrapolation and heavily depends on the choice of PDF. On the other hand, the approach is numerically robust, computationally efficient and feasible for practical application.

5.2 ECOV method – estimate of coefficient of variation

A simplified probabilistic analysis was proposed by the author [15], in which the random variation of resistance is estimated using two samples only. It is based on the idea that the random distribution of resistance, which is described by the coefficient of variation V_R, can be estimated from the mean R_m and characteristic R_k values of resistance. The underlying assumption is that random distribution of resistance is in accord with a log-normal distribution, which is typical for structural resistance. In this case it is possible to express the coefficient of variation as

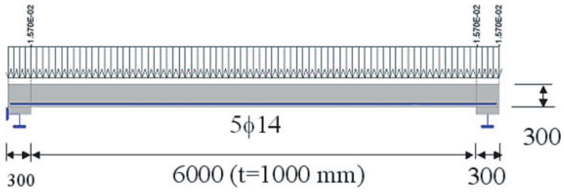
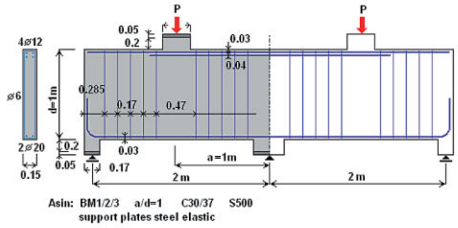
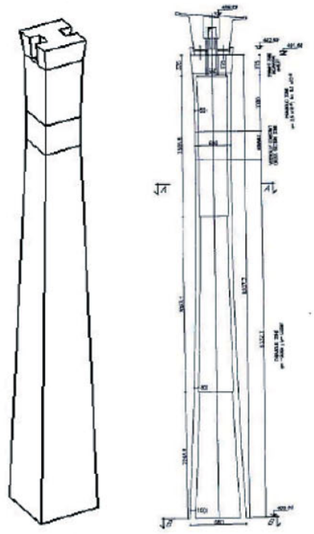
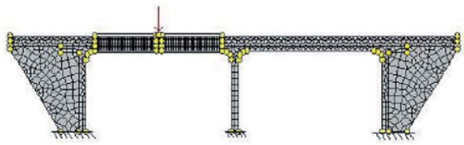
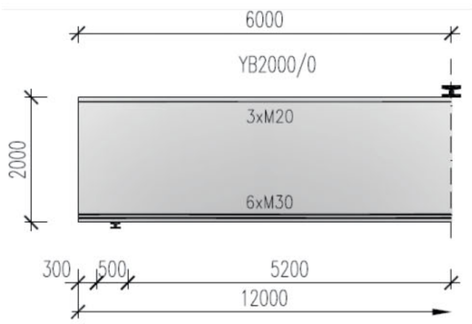
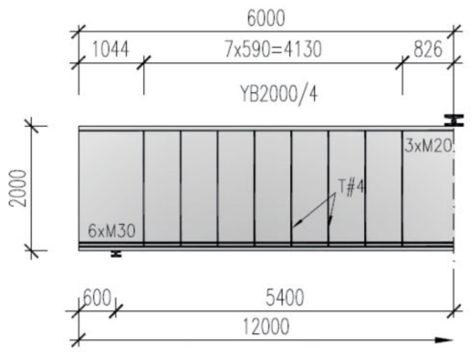
$$V_R = \frac{1}{1.65} \ln\left(\frac{R_m}{R_k}\right) \tag{5}$$

The global safety factor γ_R of resistance is then estimated using Eq. (3).

Using the typical values β = 3.8 (50 years) and α_R = 0.8 (which corresponds to the failure probability P_f = 0.001), the global resistance factor can be directly related to the estimated coefficient of variation V_R as $\gamma_R \cong \exp(3.04\ V_R)$, and the design resistance is obtained from Eq. (2).

The key element in this method is the determination of the mean and characteristic values of the resistance, R_m, R_k. It is proposed to estimate them using two separate non-linear analyses employing the mean and characteristic values of input material parameters respectively.

Table 1. Case study description

No.	Description	Scheme
1	Beam in bending	
2	Deep beam in shear	
3	Bridge pier	
	Railway bridge test	
5	Beam in shear without ties	
6	Beam in shear with ties	

The method is general and the reliability level β and distribution type can be changed if required. It reflects all types of failure. The sensitivity to random variation of all material parameters is automatically included. Thus, there is no need for special modifications to the concrete properties in order to compensate for the greater random variation of certain properties as in the next method, EN 1992-2.

A similar and refined method with more samples was proposed by *Schlune* et al. [13].

5.3 Method based on EN 1992-2

Eurocode 2 for bridges introduced a concept for verifying global safety based on non-linear analysis. Design resistance is calculated from

$$R_d = R(f_{ym}, \tilde{f}_{cm} \ldots)/\gamma_R \tag{6}$$

where f_{ym}, $\tilde{f}_{cm}$ are the mean values of the material parameters of steel reinforcement and concrete $f_{ym} = 1.1\ f_{yk}$ and $\tilde{f}_{cm} = 0.843\ f_{ck}$. Note that the mean value for concrete is reduced to account for the higher variability of the concrete property. The global factor of resistance should be $\gamma_R = 1.27$. The evaluation of the resistance function is accomplished using non-linear analysis assuming the material parameters according to the above rules.

5.4 Partial safety factors (PSFs)

The method of partial safety factors, which is used in most design codes, can be directly applied to global analysis in order to obtain the design resistance $R_d = R(f_d)$. The design values of the material parameters $f_d = f_k/\gamma_M$ are used here, where f_k are characteristic values and γ_M partial safety factors for materials.

It can be argued that design values represent extremely low material properties, which in turn do not represent real material behaviour and can thus lead to distorted failure modes. On the other hand, this method directly addresses the target design value and thus no extrapolation is involved. However, the probability of global resistance is not evaluated and is therefore not known.

6 Case study and applications

6.1 Case study for safety formats

The author has initiated investigations with the aim of comparing the various safety formats. The study comprised the six cases described in Table 1. It included a wide range of structures, including simple beam, laboratory test of a shear wall, laboratory test of a deep beam, in situ test of a real bridge and a bridge pier design case. A variety of failure modes covered ductile bending mode, brittle shear modes and a concrete compression mode. Details of this investigation can be found in [15]. A summary of the results is shown in Table 2. Three approximate methods, namely the partial safety factors (PSF) method based on the estimate of coefficient or variation of resistance (ECOV) and the method according to EN 1992-2 are evaluated. The table shows the ratio of resistances R_d found

Table 2. Case study of safety formats

	$R_d/R_d^{prob.}$		
	PSF	**ECOV**	**EN 1992-2**
Example 1 bending	1.04	1.04	0.99
Example 2 deep beam	1.02	1.04	1.0
Example 3 bridge pier	0.98	1.04	v
Example 4 bridge frame	0.99	0.96	0.92
Example 5 shear beam Y0	1.03	0.98	1.02
Example 6 shear beam Y4	0.81	1.04	0.82
average	0.98	1.01	0.95

by approximate methods to the full probabilistic analysis (which is considered as the most exact for this purpose). It should be noted that the study does not reflect the model uncertainty in a consistent way. The PSF and EN 1992-2 methods include the model uncertainty as given by the Eurocode, whereas in the ECOV and full probabilistic analysis it is not considered in order to simplify the comparison. This could explain why the average results of the ECOV method are slightly higher than the other two methods.

The study confirmed the feasibility of the approximate methods for the safety assessment. The ECOV method is preferred since it relates the safety to the resistance random variation and is considered more rational when compared with the EN 1992-2 method.

Multiple failure modes, which are typical features of reinforced concrete structures, are inherently included in the numerical models and thus they are reflected in results of analysis and resistance variability. Therefore, the approximate methods of safety verification are generally applicable in design. In significant cases, if justified by the consequences of failure, a full probabilistic analysis should be applied.

6.2 Large shear beam

To illustrate this, two applications of design verification by non-linear analysis will be shown. The first example applies the safety formats discussed above to a large beam tested in the laboratory by *Collins* et al. [12] and already mentioned in Fig. 8. Its size is large and exceeds usual beam dimensions (span = 12 m, depth = 2 m). The shear failure is apparently influenced by its large size and is very brittle. The comparison of resistances obtained by various safety formats is shown in Fig. 10, which also shows the values of design resistance obtained with EN 1992-1 and ACI 318.

This case reveals two remarkable features of numerical simulation. First, a refined constitutive modelling based on fracture mechanics can capture the size effect of brittle shear failure and provide a safer model of resis-

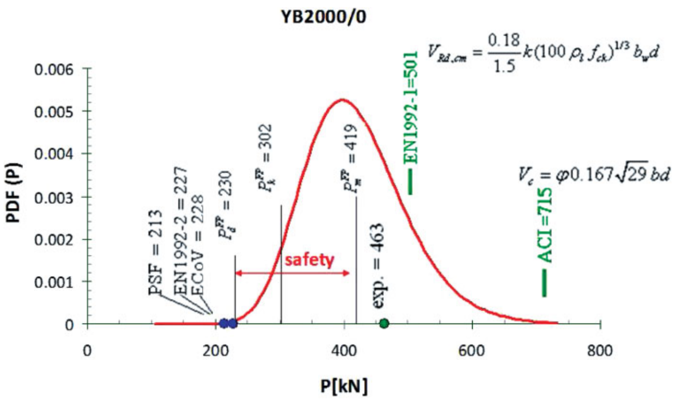


Fig. 10. Comparison of safety margins in shear failure

Fig. 11. Bridge under construction, built using the balanced cantilever method

tance. Second, the global safety formats offer consistent safety margins for the design verification.

6.3 Box girder bridge

The bridge over the River Berounka on the recently opened ring road around the city of Prague was designed with the help of numerical simulation. It is a box girder integral structure with complex geometry curved in three dimensions and supported on slender piers. During construction stages when the girder was not yet integrally connected with other spans, it was very sensitive to stability conditions, see Fig. 11. The safety of construction phases was verified by numerical simulation and global safety format.

For illustration only, a result of the load case with proportionally increased uniform load is presented in Fig. 12, showing cracks and plastic deformations. This helps to identify a mode of failure reached at the ultimate limit state. The evidence of structural resistance is provided by a load–displacement diagram (Fig. 13). The relative load on the vertical axis is a non-dimensional overloading parameter representing the global safety factor γ_R from Eq. (2). In this case the analysis confirmed the safety factor γ_R = 1.7, which is well above the value of 1.27 required by the code. The global safety factors obtained for the other load histories due to the construction phases of the balanced cantilever method were 6.2 for wind action and 5.5 for formwork action during the cantilever construction. A sufficient safety margin was confirmed for all stages of construction.

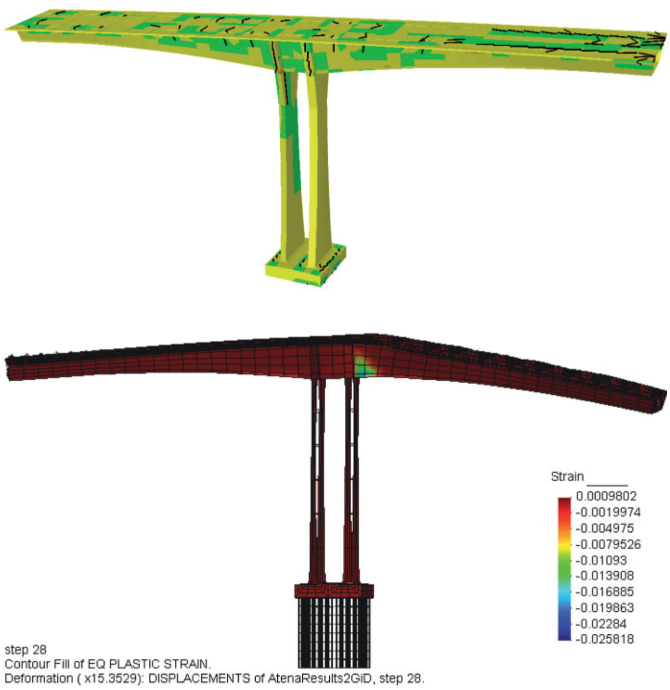


Fig. 12. Cracks and plastic strains at maximum uniform load

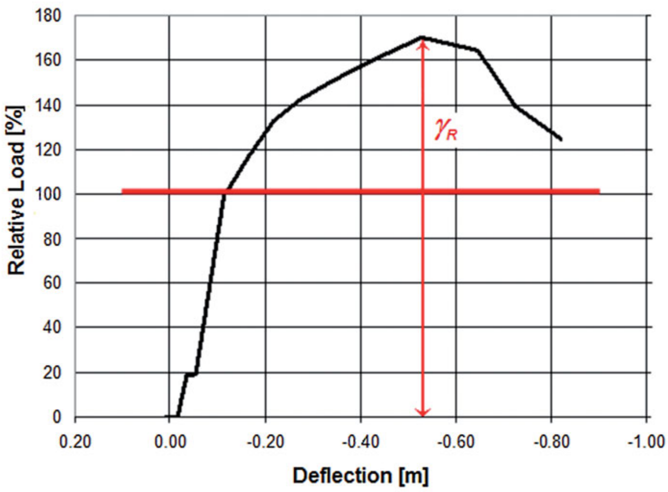


Fig. 13. Load-displacement diagram for bridge during construction

The shape of a descending branch in the load-displacement diagram provides additional information about the ductile nature of the failure, which is an important measure of robustness. The case observed indicates a relatively brittle behaviour, which in this case is due to a compressive failure of the concrete, which occurs in the box girder following cable yielding and excessive rotation, and in some load cases in the concrete of the pier. More details can be found in [1].

7 Closing remarks

Verification by numerical simulation is a powerful tool for the design of concrete structures. It extends the range of application beyond the scope of engineering methods based on the elastic distribution of internal forces and cross-section safety check. Owing to its general approach, it overcomes the limits of standard design based on beams and columns. On the other hand, it introduces potentially higher model uncertainties. Therefore, the model validation becomes an important requirement for its application in engineering practice.

fib Model Code 2010 outlines the framework of limit state verification by numerical simulations and introduces the global safety formats suggested for this purpose.

Further research is needed in order to improve the guide for the validation of numerical models and the classification of model uncertainties.

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Vladimir Cervenka,
President, Cervenka Consulting
Na Hrebenkach 55, 15000 Prague 5,
Czech Republic
Tel. +420 220 610 018,
e-mail: vladimir.cervenka@cervenka.cz