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Title:

**Large Deflections**

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## 1 Introduction

This report describes the resistance of 2D beam and 3D slab structures due to large deflections. The aim of the study is to model situations resulting from loading under sever conditions, in which the second order effect of deformations contributes to increased load bearing capacity due to membrane resistance of reinforcement. Such situations can be caused by failure of supports due to an accident, after which the load is transferred by tensile capacity of the deformed structure.

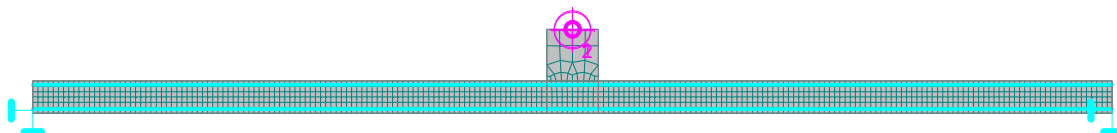
## 2 Beam

### 2.1 Geometry, supports and loading, properties

Geometry of the RC beam is shown in Fig. 2-1. Beam cross section is 0.2 x 0.4m. Length is 8.4m. Reinforcement is by 2 $\varnothing$ 20 in bottom and in top, with the cover 25mm. Fixed supports are at both ends. Loading is applied in the middle on the top of block, 0.4m wide. Concrete is C30 (cube strength 30MPa, cylinder strength 25.5 MPa), reinforcement yield strength 500 MPa.

Loading is applied by prescribed displacement at the center on the top of a block 0.4m wide. Block is considered from elastic material.

Material model for concrete is SBETA 2D, model for reinforcement is bilinear elastic-perfectly plastic, with unlimited plasticity.



**Fig. 2-1 Beam. Reinforcement, FE mesh, fixed supports.**

Load is applied by increments of 5mm in each load step. Large deformation and geometrical non-linearity (in strain-displacement relations) are utilized in the analysis. Analysis is performed with ATENA 2D Egr. Isoparametric 4-node quadrilateral element is used for concrete and embedded truss element for reinforcement.

### 3 Results

#### 3.1 Load-displacement response

Load-displacement diagram is shown in Fig. 3-1. The resistance is increasing until the local extreme at **Stage A** ( $P=78.5$  kN,  $d=80$  mm), followed by softening to the **Stage B** ( $P=58.5$  kN,  $d=240$  mm) and hardening to the **Stage C** ( $P=318.1$  kN,  $d=1050$  mm).

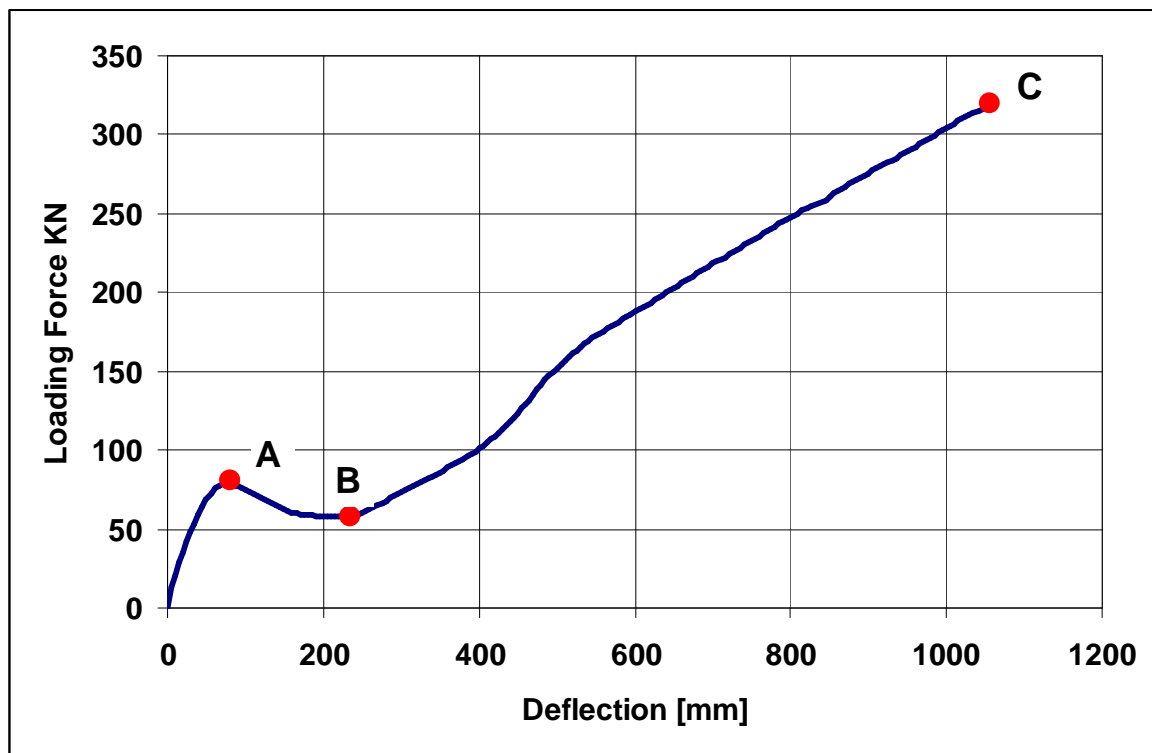


Fig. 3-1 Load displacement diagram of RC beam.

### 3.2 Deformation, stresses and strains in load stages

#### 3.2.1 Stage A

Step 20,



Fig. 3-2 Deformations in real scale 1:1.

Step 20,

Cracks: in elements, <1.000E-04; ...), opening: <-6.032E-06;1.111E-03>[m], Sigma\_n: <0.000E+00;5.205E+00>[MPa], Sigma\_T: <

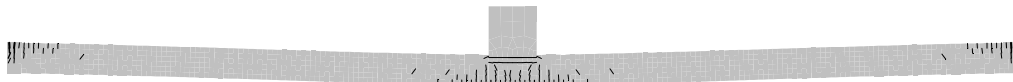


Fig. 3-3 Crack pattern (crack width > 0.1mm)

Step 20,

Skalárý: barevný poechod, Basic material, in nodes, Principal Stress, Min., <-2.589E+01;3.110E-01>[MPa]

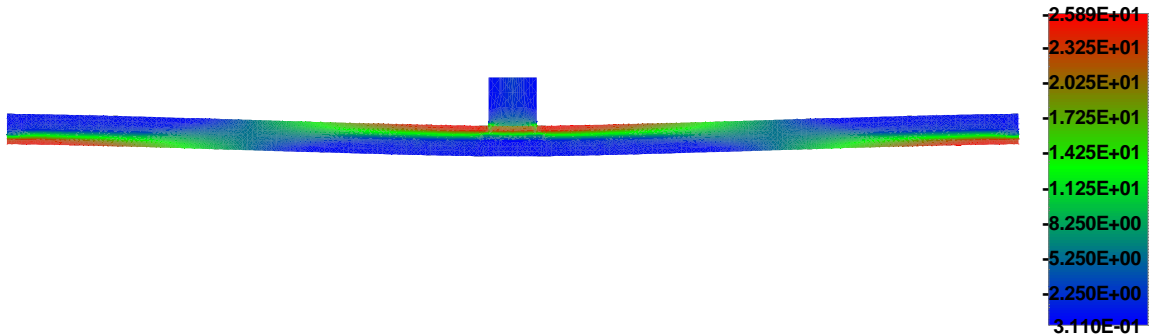


Fig. 3-4 Principal compressive stresses. Maximum effect of arch action in concrete.

Step 20,

Reinforcements: Stress, Sigma xx, <-5.000E+02;5.000E+02>[MPa]

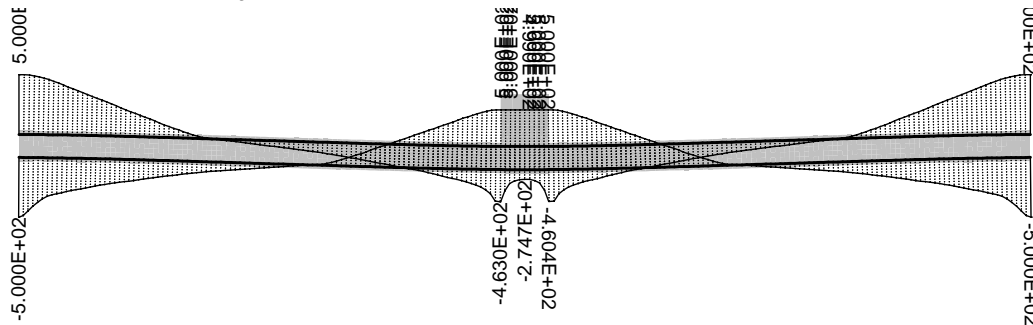


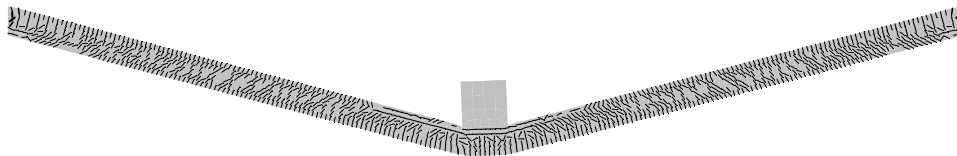
Fig. 3-5 Stresses in reinforcing bars.



### 3.2.3 Stage C

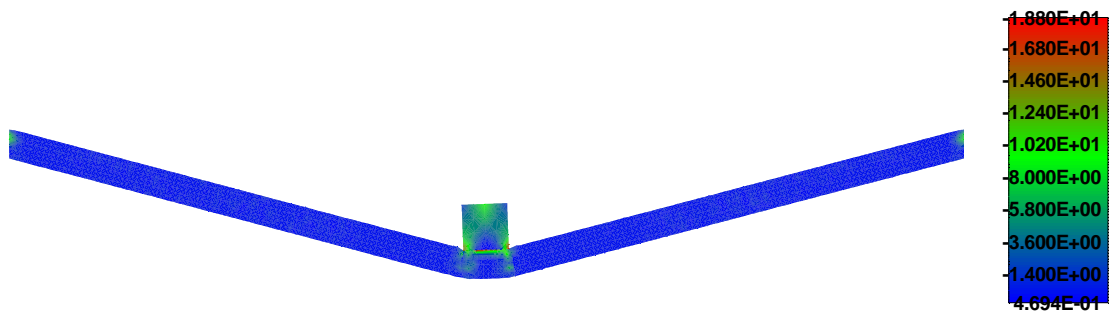
**Fig. 3-10 Deformations in real scale 1:1.**

Step 210,  
Cracks: in elements, <1.000E-04; ...), opening: <-8.354E-03;3.110E-02>[m], Sigma\_n: <-1.917E+01;1.202E+00>[MPa], Sigma\_T: <



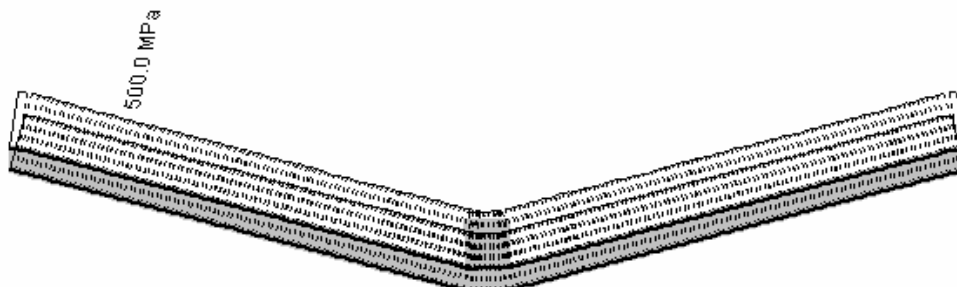
**Fig. 3-11 Crack pattern (crack width > 0.1mm)**

Step 210,  
Skalárý: barevný poechod, Basic material, in nodes, Principal Stress, Min., <-1.880E+01;4.694E-01>[MPa]



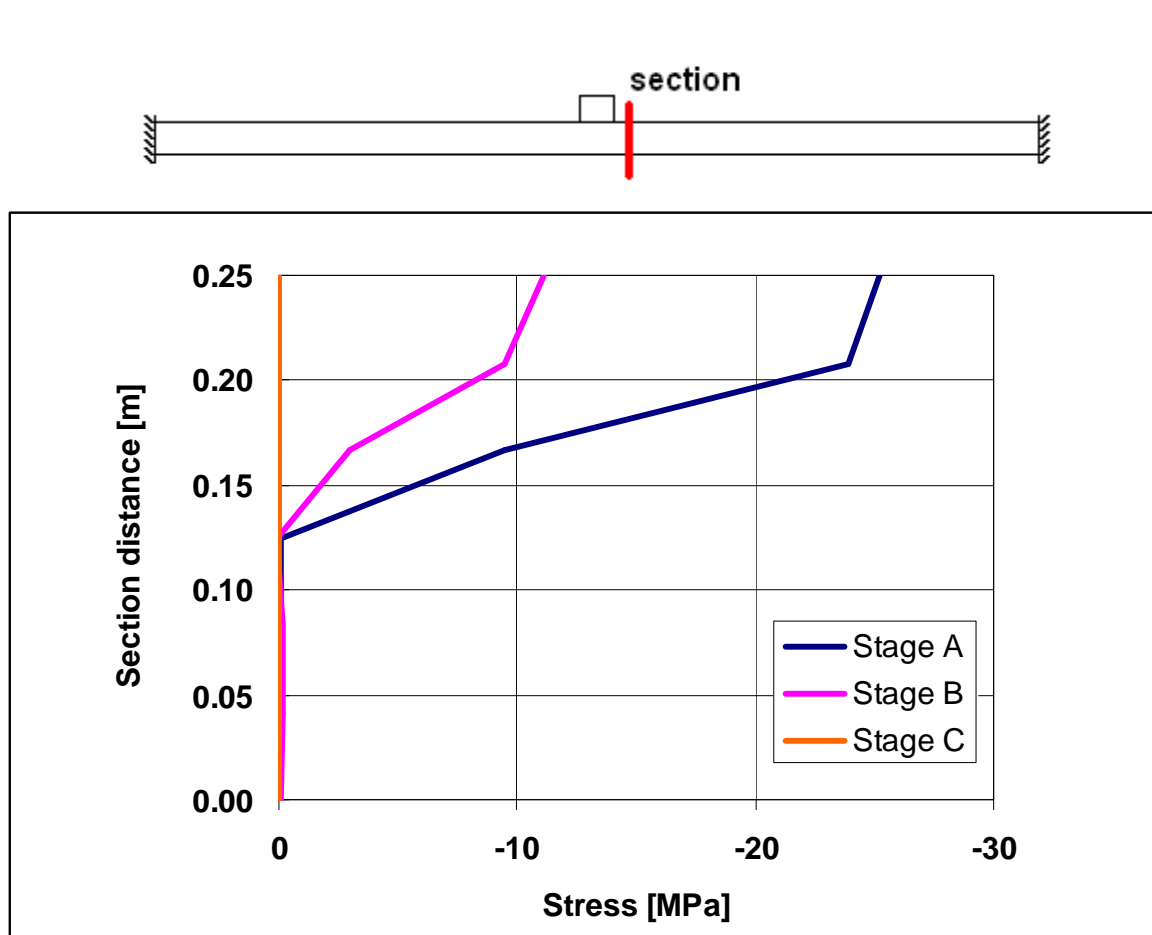
**Fig. 3-12 Principal compressive stresses. Nonexistent arch action in concrete.**

Step 210,  
Reinforcements: Stress, Sigma\_xx, <4.762E+02;5.000E+02> [MP a]



**Fig. 3-13 Stresses in reinforcing bars. All stresses are tensile at yielding level 500 MPa.**





**Fig. 3-15 Stress distribution in concrete section at different load stages. The location of section in the beam shown above.**

Stress distribution in the cross section of the beam near the center shown in Fig. 3-15 illustrate the changing function of concrete in load carrying resistance of beam. In the initial arching action at Stage A the stress in the compression zone of concrete are close to its compressive strength. By continuing deformation the stress is reduced in Stage B. In final Stage C the concrete is unloaded and all resistance is due to tensile action of reinforcement in changed geometry.

### 3.2.4 Limited ductility of reinforcement

The same beam was also analyzed with limited plastic strain of reinforcement 12%. The diagrams for limited and unlimited steel ductility are compared in . The rupture of reinforcement is indicated by sudden drops in resistance.

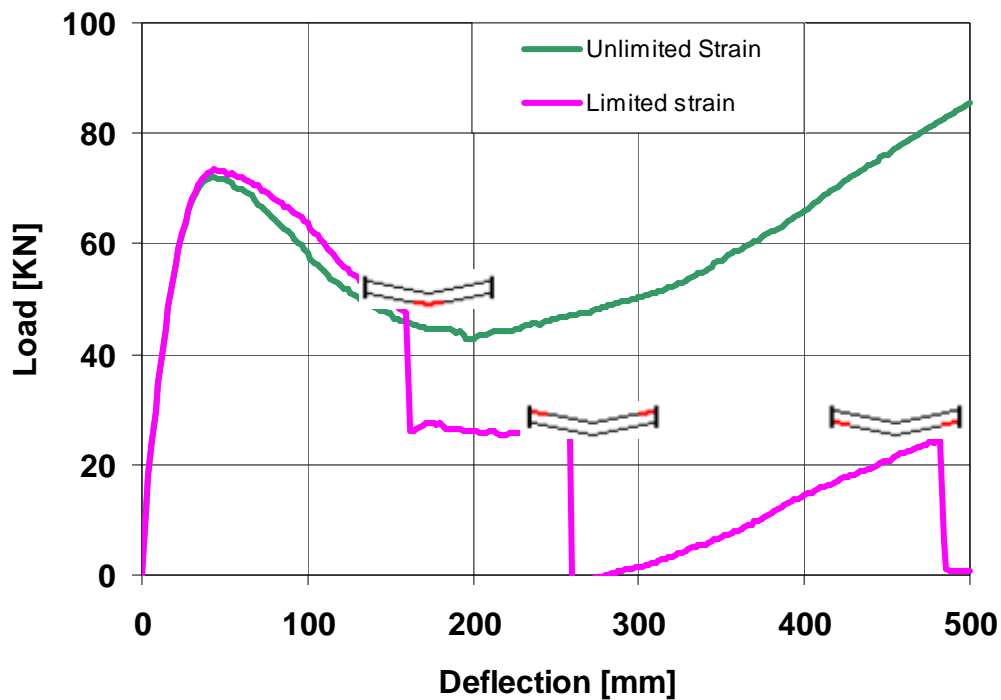
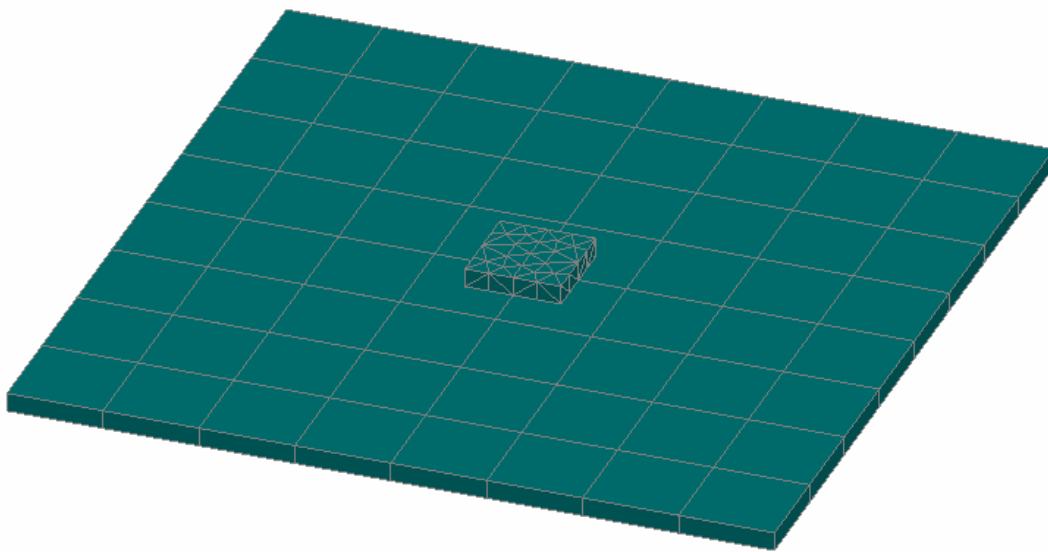


Fig. 3-16 Load-displacement diagrams for reinforcement with limited ductility.

## 4 Slab

Reinforced concrete slab has thickness 0.2m, dimensions are 8x8m. It is fixed on all sides (fixed rotation and displacements) and loaded by prescribed displacement in the slab center through a solid block 1x1x0.2 m. See Fig. 4-1. Reinforcement is by 12 bars with spacing 125mm in two orthogonal directions X, Y at top and bottom of the slab. Cover of the reinforcement is 20mm. Concrete properties are based on the cube strength 30 MPa (cylinder strength 25.5 MPa.) Reinforcement yield stress is 500 MPa.



**Fig. 4-1 RC Slab.**

Material model for concrete used in analysis is the fracture-plastic model (ATENA 3D Nonlinear Cementitious 2) and bi-linear elastic-plastic model for reinforcement.

Ahmad high order shell element is used for the slab. 3x integration scheme, with 4 layers at each integration point. Smeared reinforcement is considered in layers for each position and direction of bars.

Loading is applied as prescribed displacement at the center at increments of 5mm in a load step. The loading force is found as a reaction corresponding to the prescribed displacement. This solution method enables to determine a softening response of resistance.

## 4.1 Load-displacement response

Load-displacement diagram is shown in Fig. 4-2. The resistance is increasing until the local extreme at Stage A ( $P=1749$  kN,  $d=104$  mm), followed by softening in Stage B ( $P=1474$  kN,  $d=220$  mm) and hardening in Stage C ( $P=2426$  kN,  $d=501$  mm).

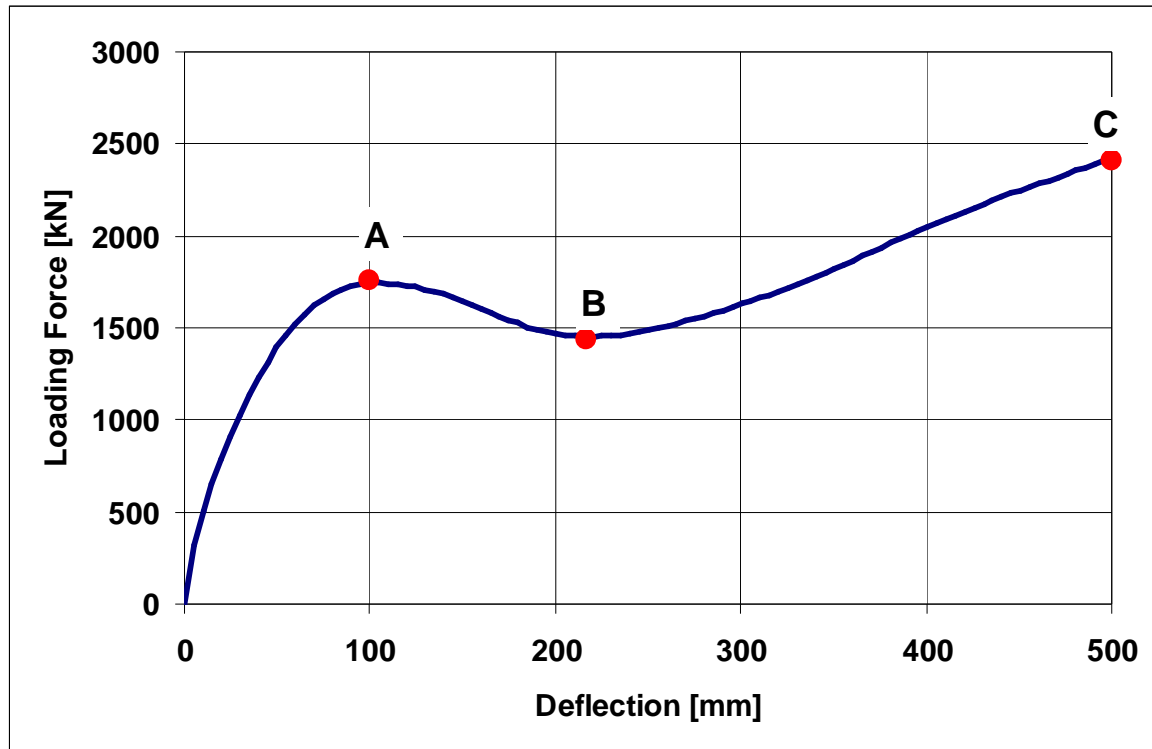


Fig. 4-2 Load displacement diagram of RC beam.

## 4.2 Deformation, stresses and strains in load stages of slab

### 4.2.1 Stage A

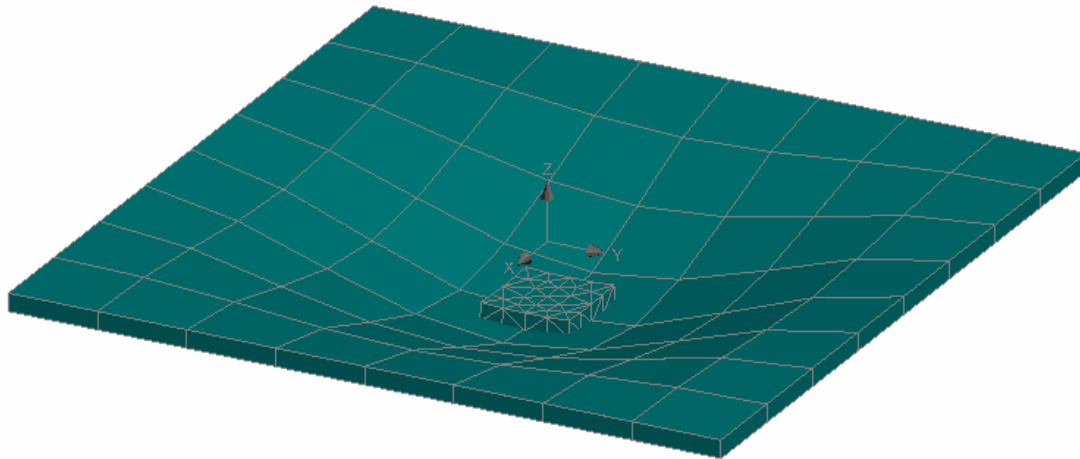


Fig. 4-3 Deformations enlarged 10x.

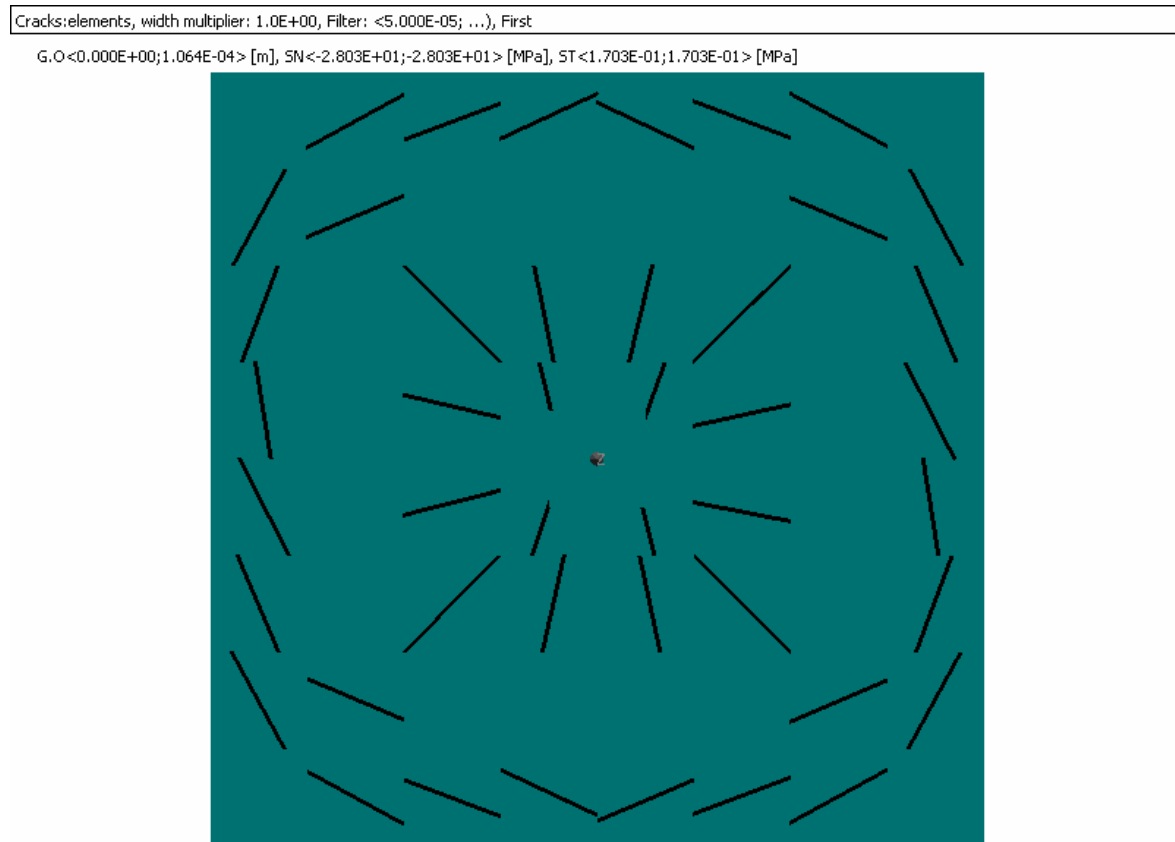


Fig. 4-4 Crack pattern showing cracks on both surfaces (top and bottom).

Scalars:iso-areas, in nodes, Crack Width, Cod1, G. <-1.133E-04;3.857E-04> [m]

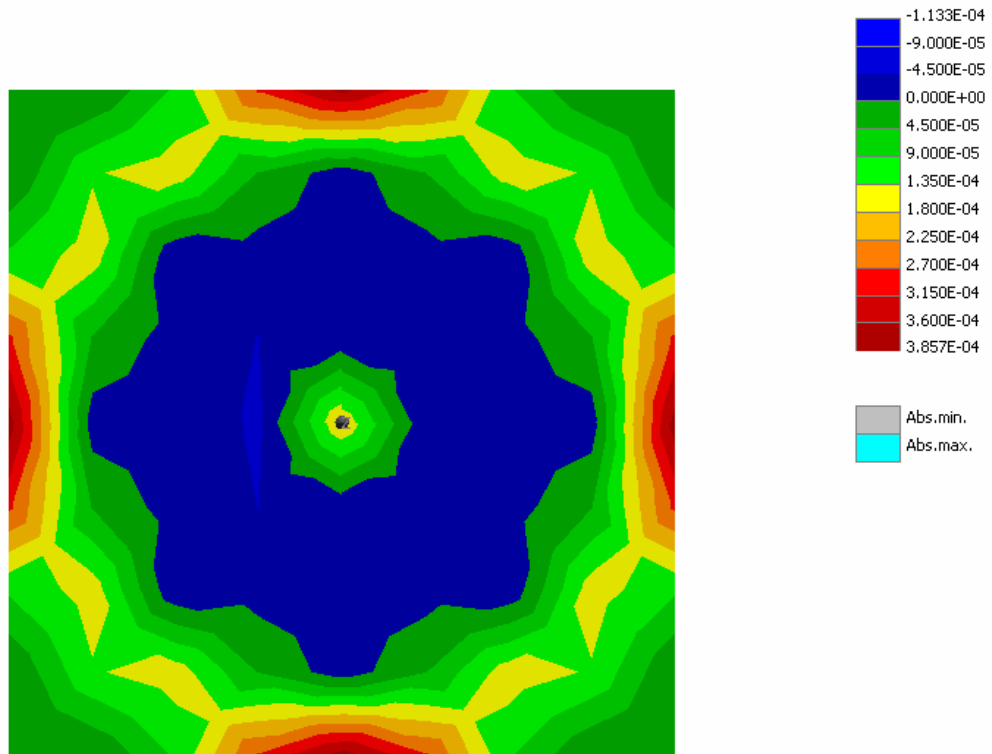


Fig. 4-5 Crack widths on top surface

Scalars:iso-areas, in nodes, Principal Stress, Min., A. <-2.883E+01;5.122E-03> [MPa], G. <-8.054E+01;5.122E-03> [MPa]

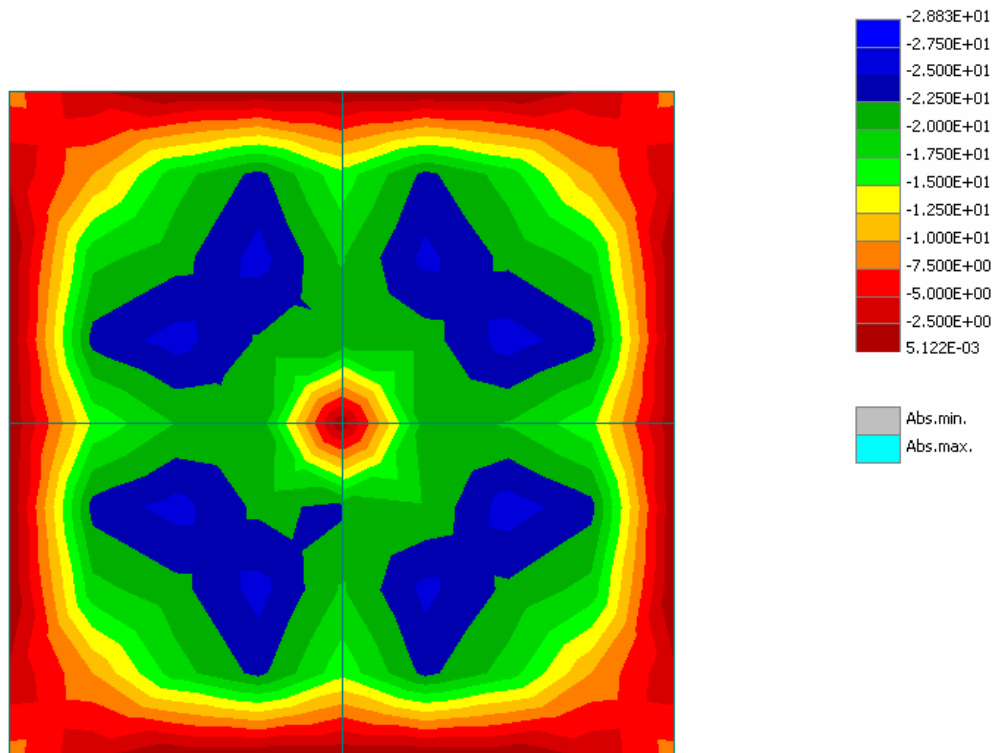


Fig. 4-6 Principal compressive stresses on the top surface.

Scalars:iso-areas, in nodes, Stress (r1), Sigma xx, A. <-3.493E+02;5.000E+02> [MPa], G. <-3.493E+02;5.000E+02> [MPa]

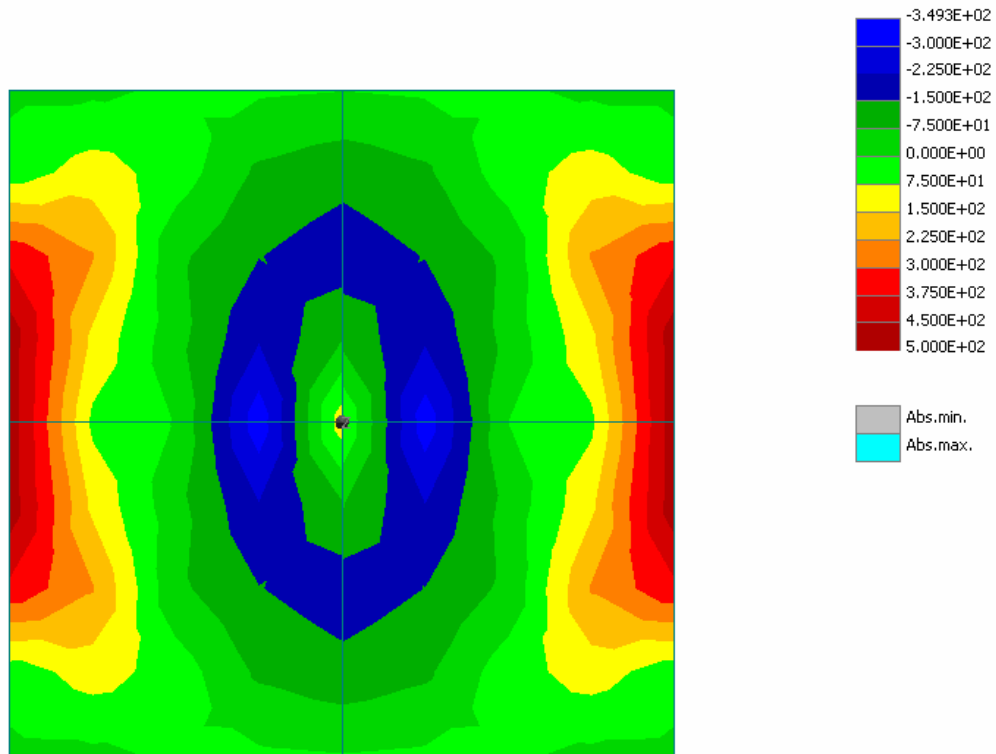


Fig. 4-7 Stresses in reinforcing bars near to top surface (X - horizontal dir.).

Scalars:iso-areas, in nodes, Stress (r4), Sigma xx, A. <-5.000E+02;5.000E+02> [MPa], G. <-5.000E+02;5.000E+02> [MPa]

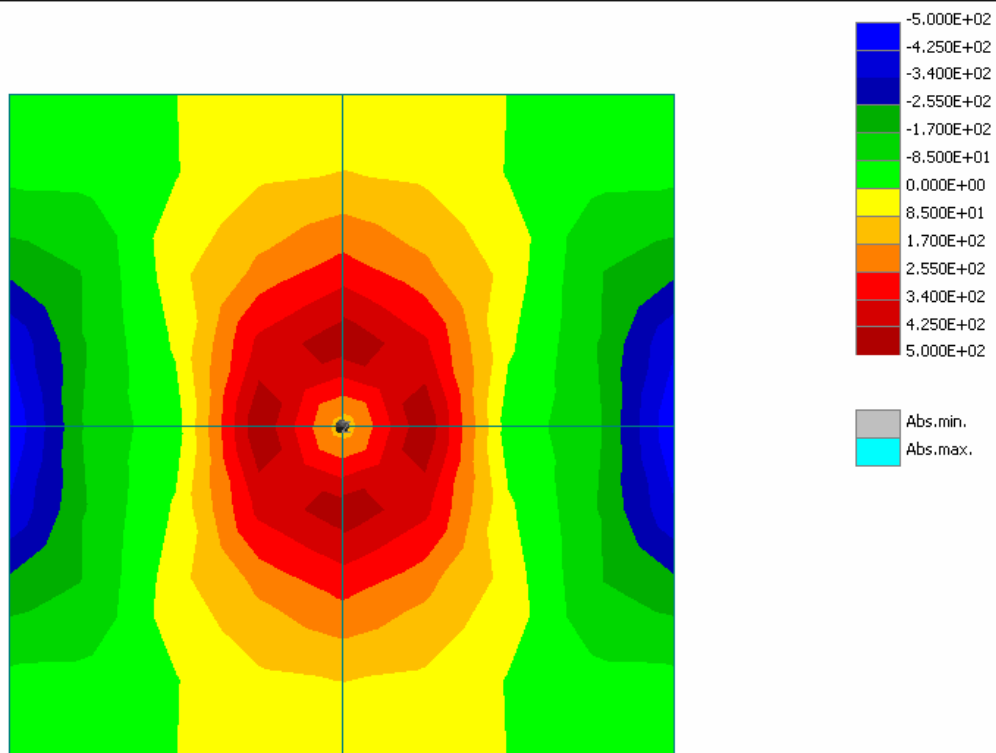


Fig. 4-8 Stresses in reinf. bars near to bottom surface (X - horizontal dir.).

### 4.2.2 Stage B

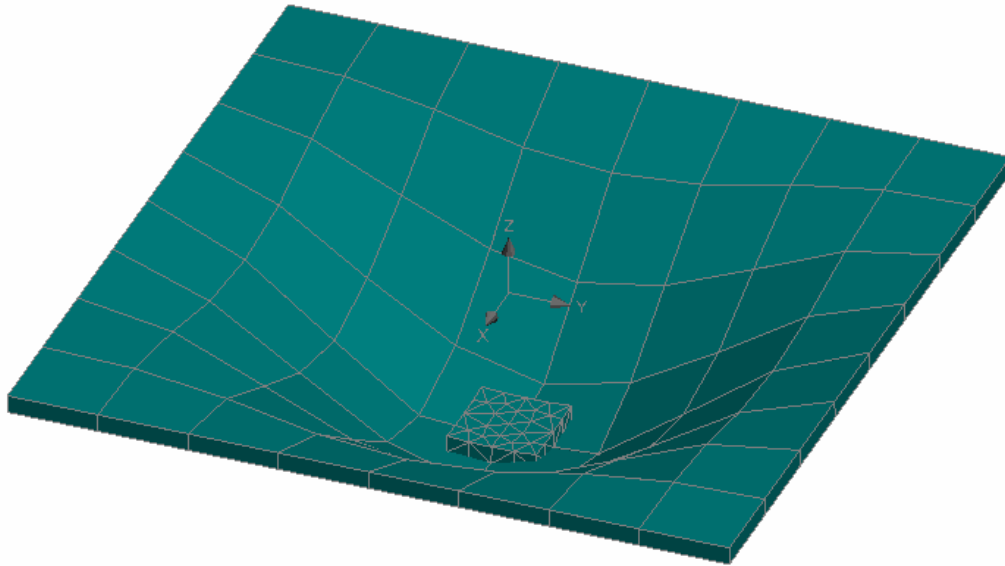


Fig. 4-9 Deformations enlarged 10x.

Cracks:elements, width multiplier: 1.0E+00, Filter: <5.000E-05; ...), First

G.O<0.000E+00;1.868E-04> [m], SN<-1.976E+01;-1.976E+01> [MPa], ST<1.551E-01;1.551E-01> [MPa]

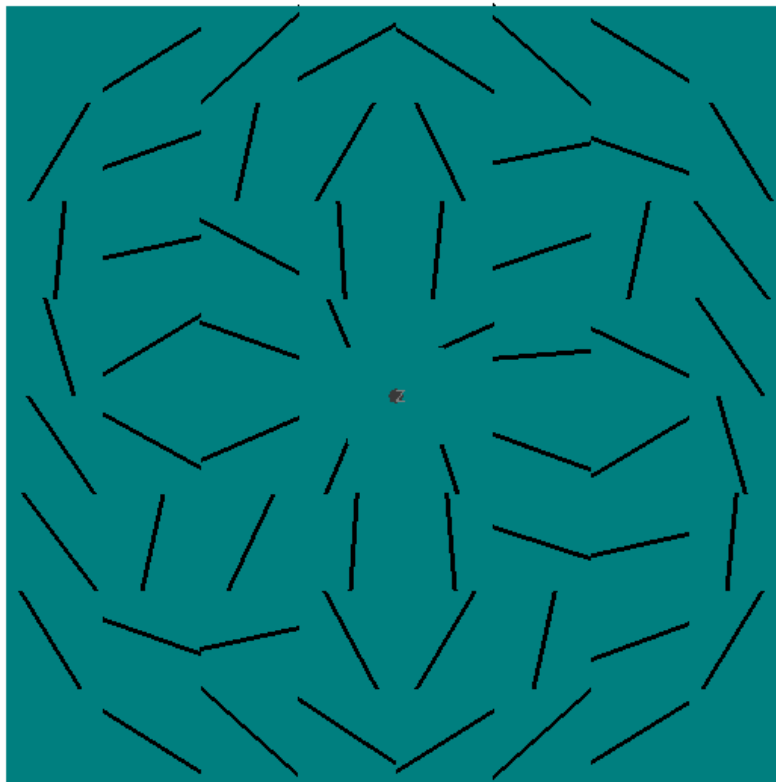


Fig. 4-10 Crack pattern showing cracks on both surfaces (top and bottom).

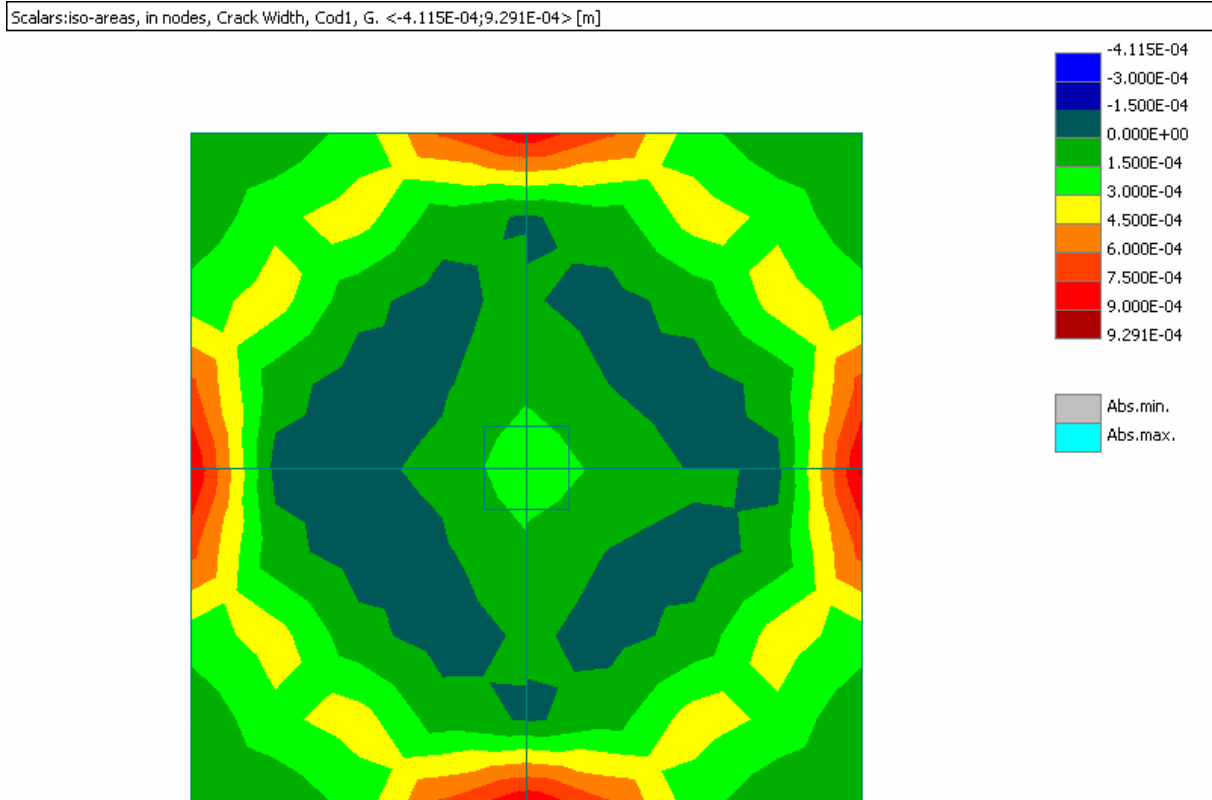


Fig. 4-11 Crack widths on top surface

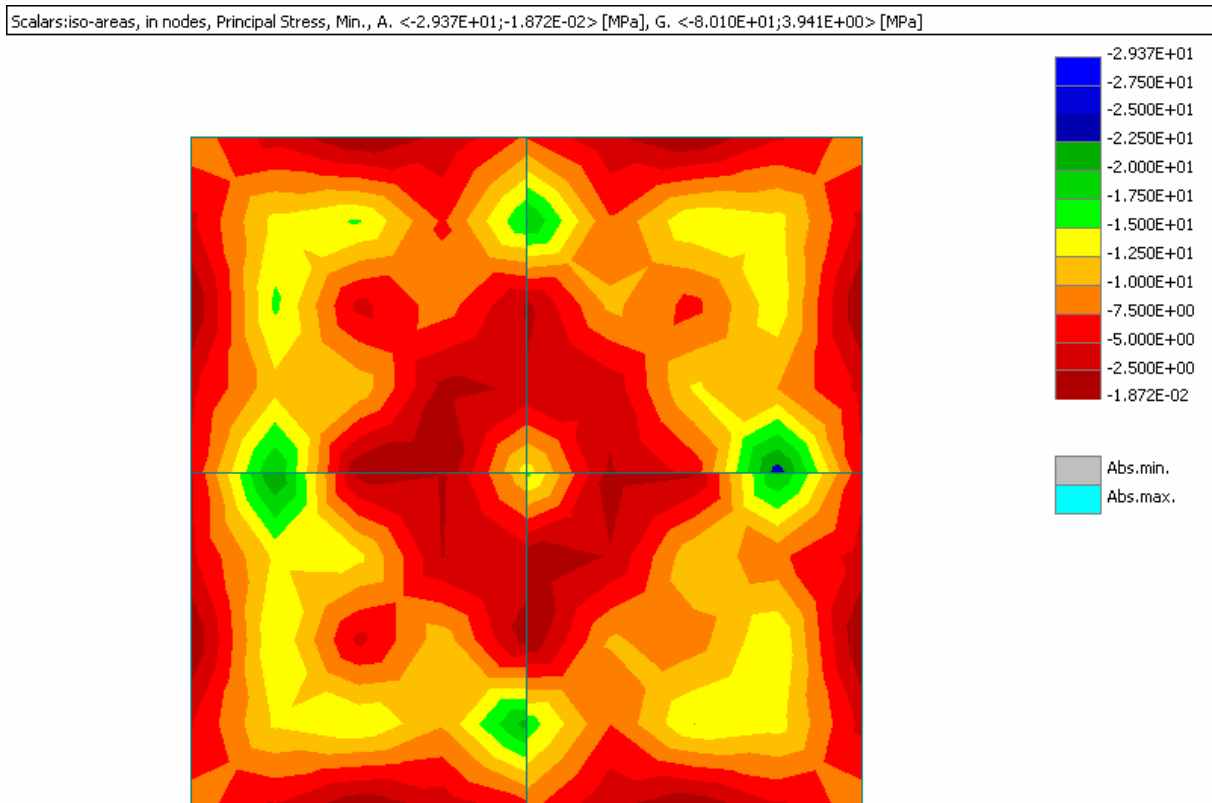


Fig. 4-12 Principal compressive stresses on the top surface.

Scalars:iso-areas, in nodes, Stress (r1), Sigma xx, A. <-3.493E+02;5.000E+02> [MPa], G. <-3.493E+02;5.000E+02> [MPa]

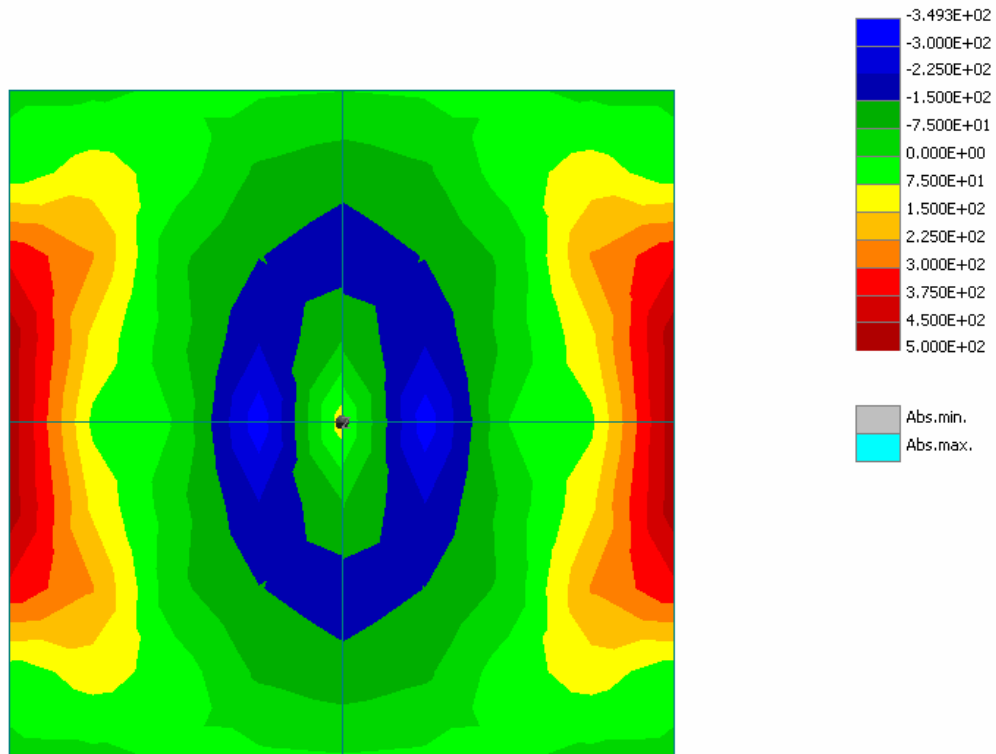


Fig. 4-13 Stresses in reinf. bars near top surface (X - bars in horizontal dir.).

Scalars:iso-areas, in nodes, Stress (r4), Sigma xx, A. <-5.000E+02;5.000E+02> [MPa], G. <-5.000E+02;5.000E+02> [MPa]

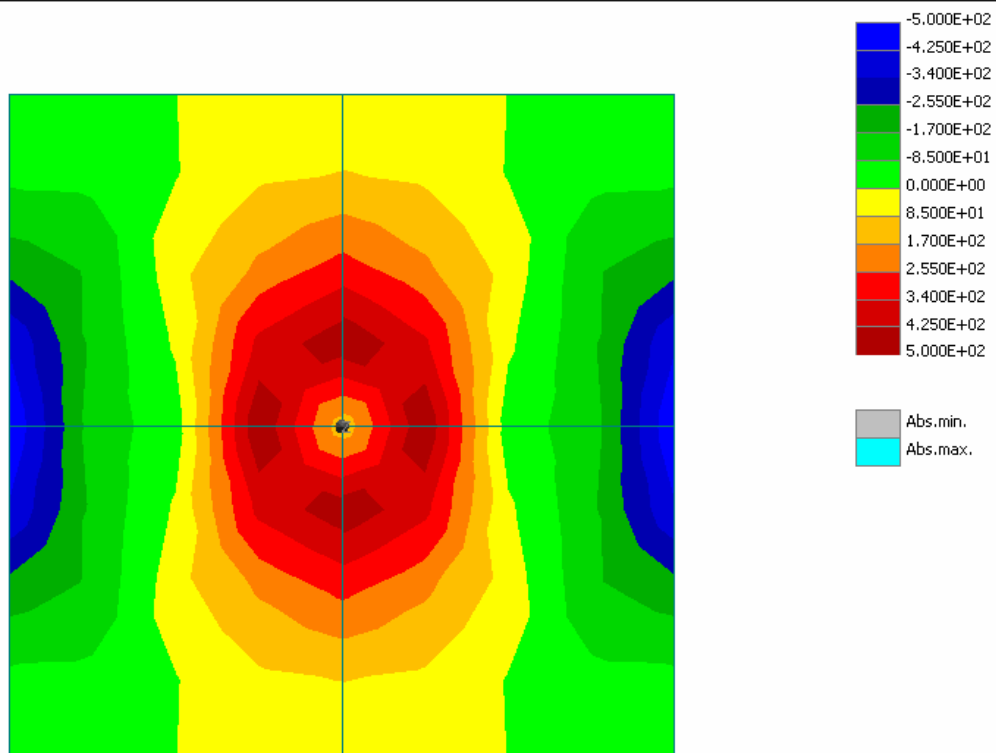
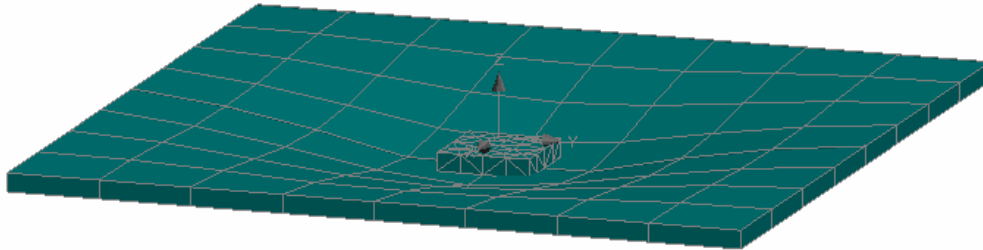


Fig. 4-14 Stresses in reinf. bars near bottom surface (X - horizontal dir.).

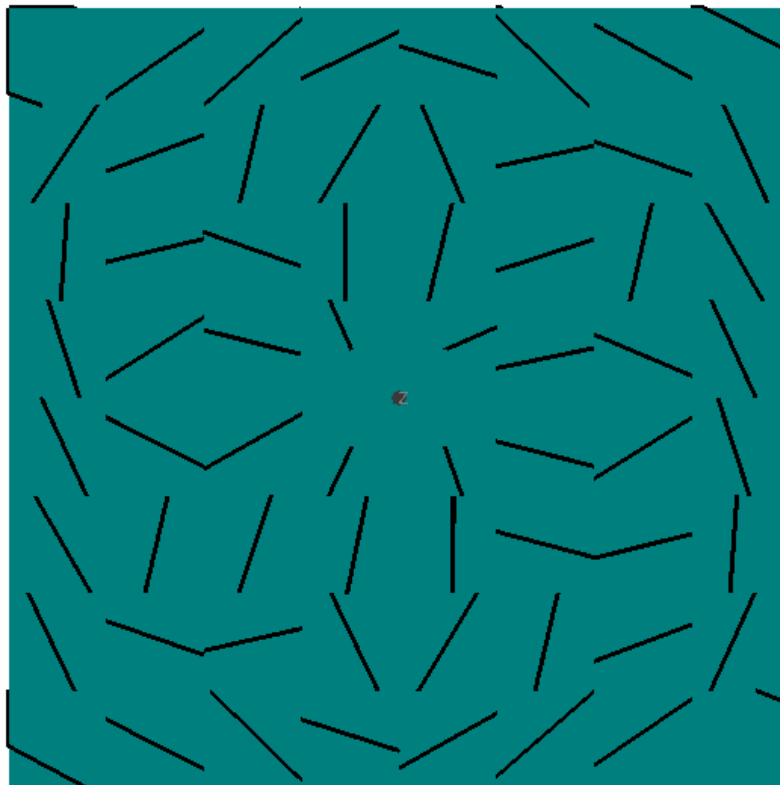
### 4.2.3 Stage C



**Fig. 4-15 Deformations in real scale 1:1.**

Cracks:elements, width multiplier: 1.0E+00, Filter: <5.000E-05; ...), First

G.O<6.169E-06;1.255E-03> [m], SN<-3.792E+00;-3.792E+00> [MPa], ST<2.090E-01;2.090E-01> [MPa]



**Fig. 4-16 Crack pattern showing cracks on both surfaces (top and bottom).**

Scalars:iso-areas, in nodes, Crack Width, Cod1, A. <-2.084E-03;3.019E-03> [m], G. <-2.084E-03;3.019E-03> [m]

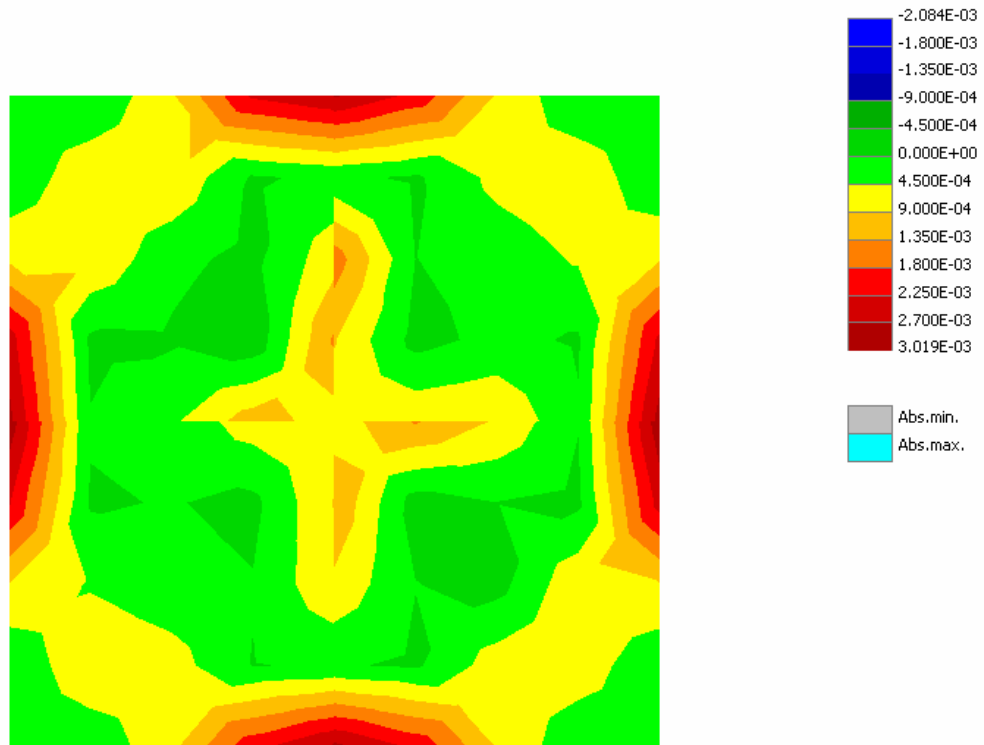


Fig. 4-17 Crack widths on top surface

Scalars:iso-areas, in nodes, Principal Stress, Min., A. <-3.076E+01;1.483E-02> [MPa], G. <-1.160E+02;1.990E+00> [MPa]

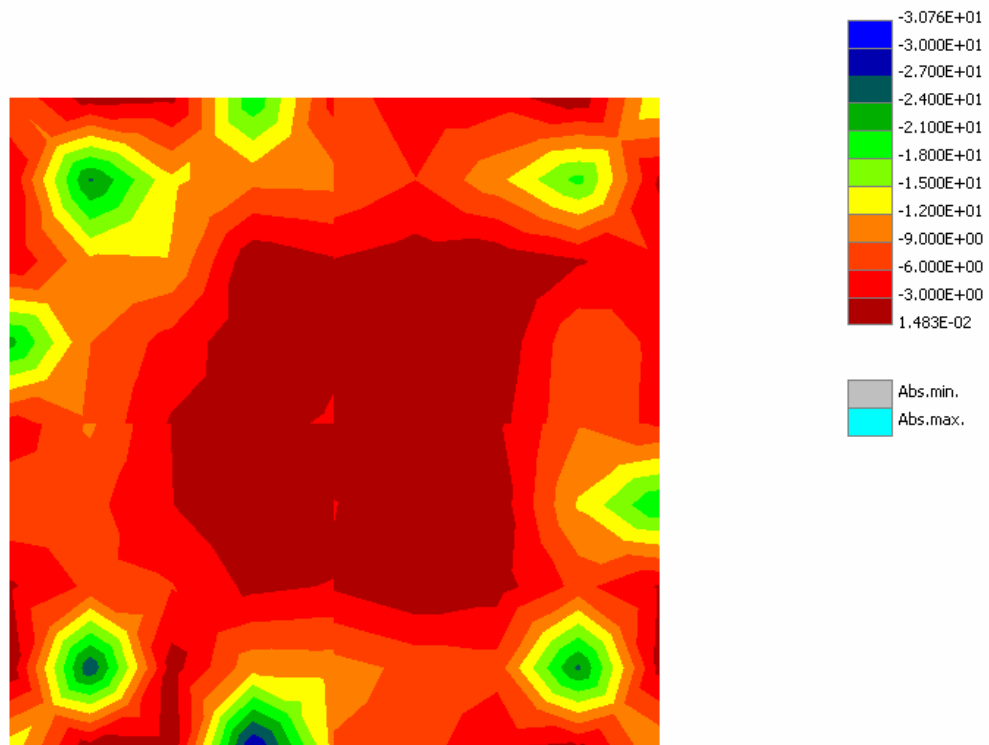


Fig. 4-18 Principal compressive stresses on the top surface.

Scalars:iso-areas, in nodes, Stress (r1), Sigma xx, A. <-5.000E+02;5.000E+02> [MPa], G. <-5.000E+02;5.000E+02> [MPa]

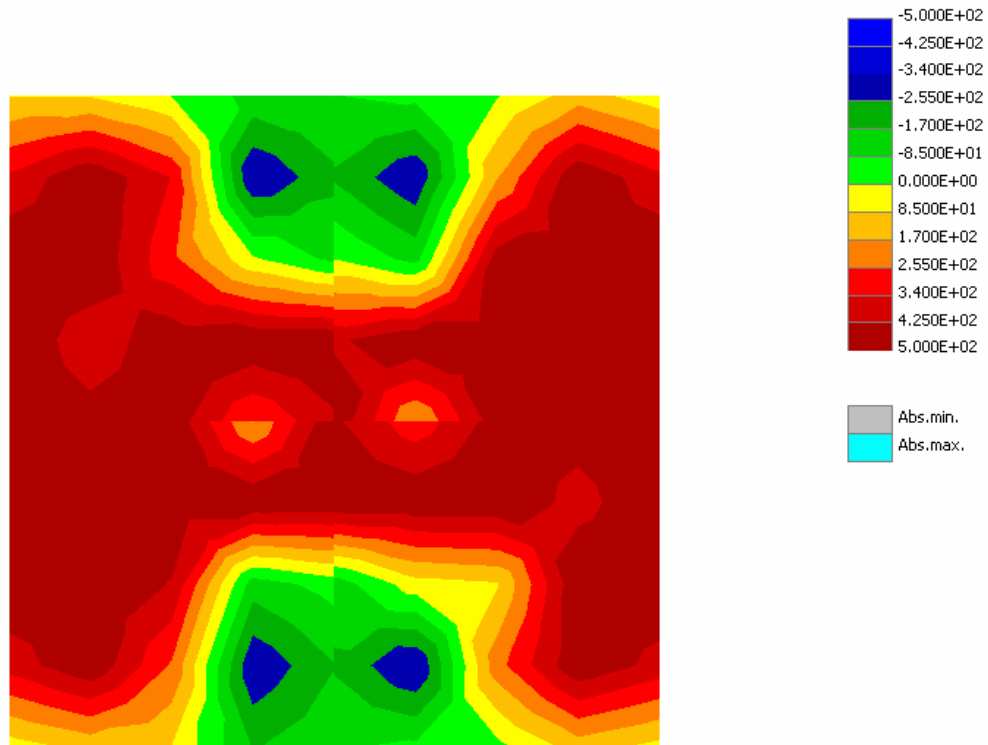


Fig. 4-19 Stresses in reinf. bars near top surface (X - in horizontal dir.).

Scalars:iso-areas, in nodes, Stress (r2), Sigma xx, A. <-5.000E+02;5.000E+02> [MPa], G. <-5.000E+02;5.000E+02> [MPa]

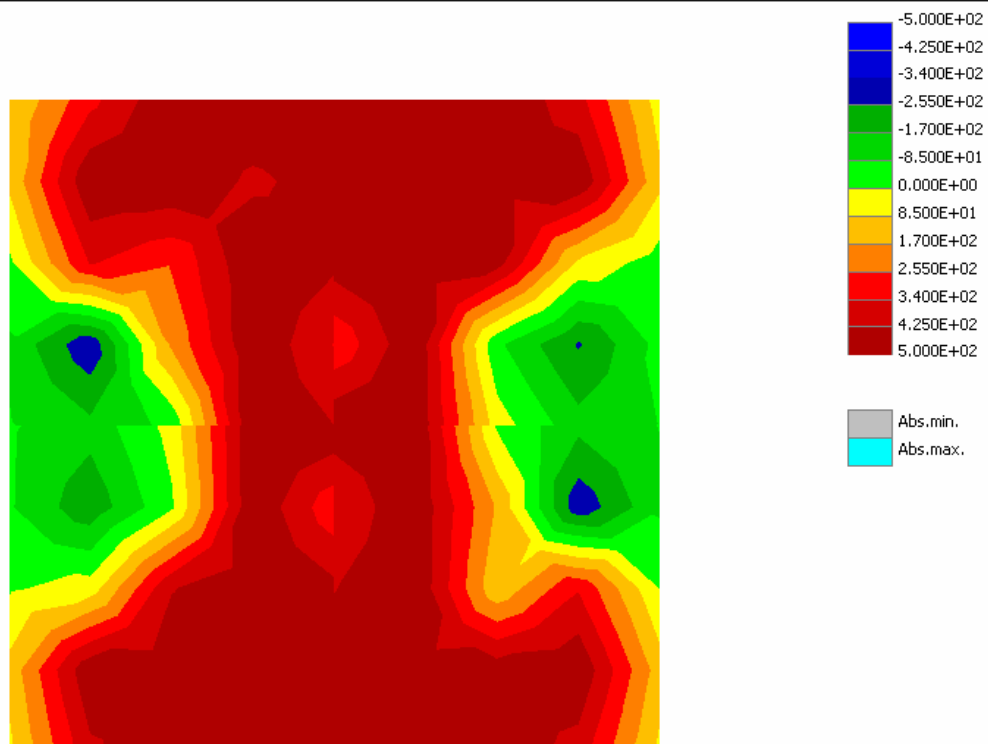


Fig. 4-20 Stresses in reinf. bars near top surface (Y - in vertical dir.).

Scalars:iso-areas, in nodes, Stress (r4), Sigma xx, A. <-5.000E+02;5.000E+02> [MPa], G. <-5.000E+02;5.000E+02> [MPa]

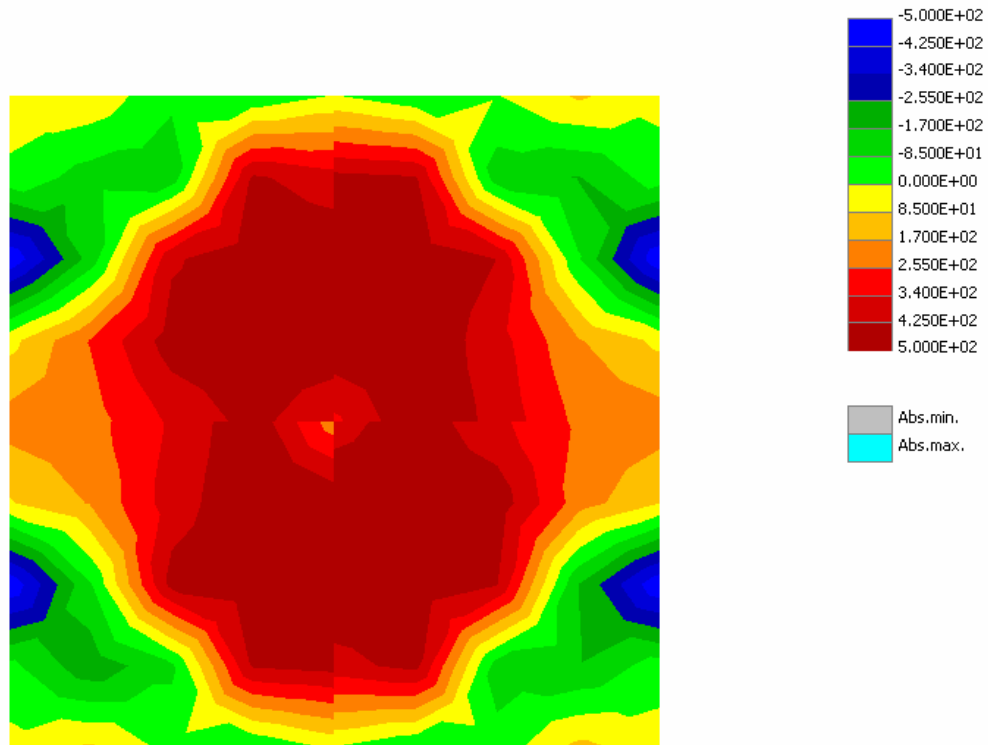


Fig. 4-21 Stresses in reinf. bars near bottom surface (X - horizontal dir.).

Scalars:iso-areas, in nodes, Stress (r3), Sigma xx, A. <-4.394E+02;5.000E+02> [MPa], G. <-4.394E+02;5.000E+02> [MPa]

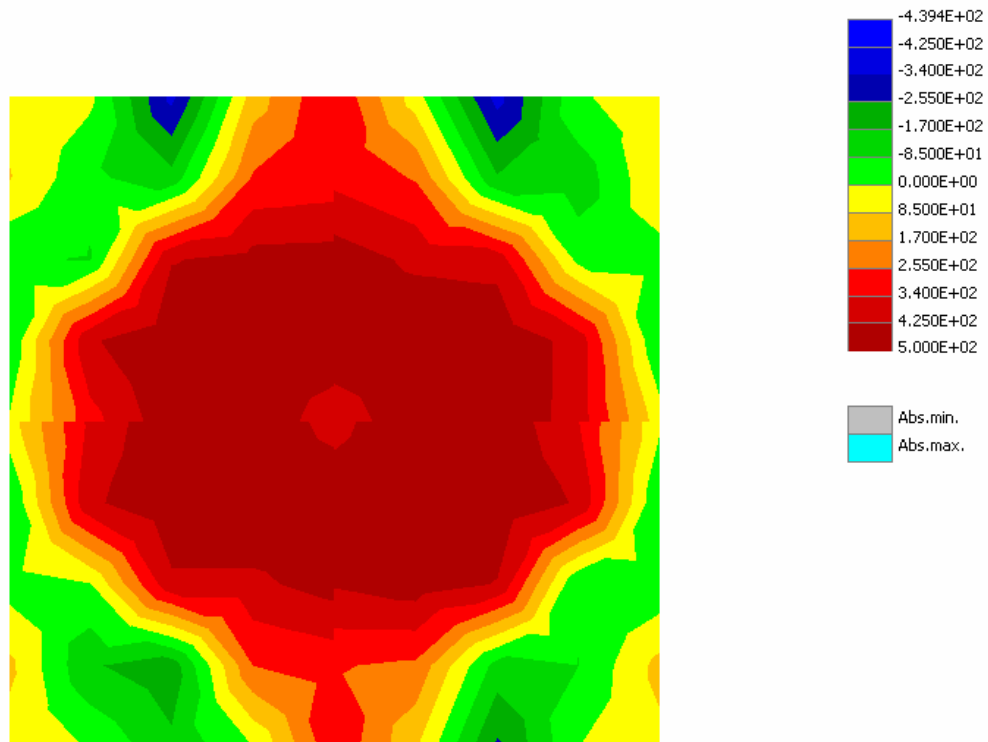
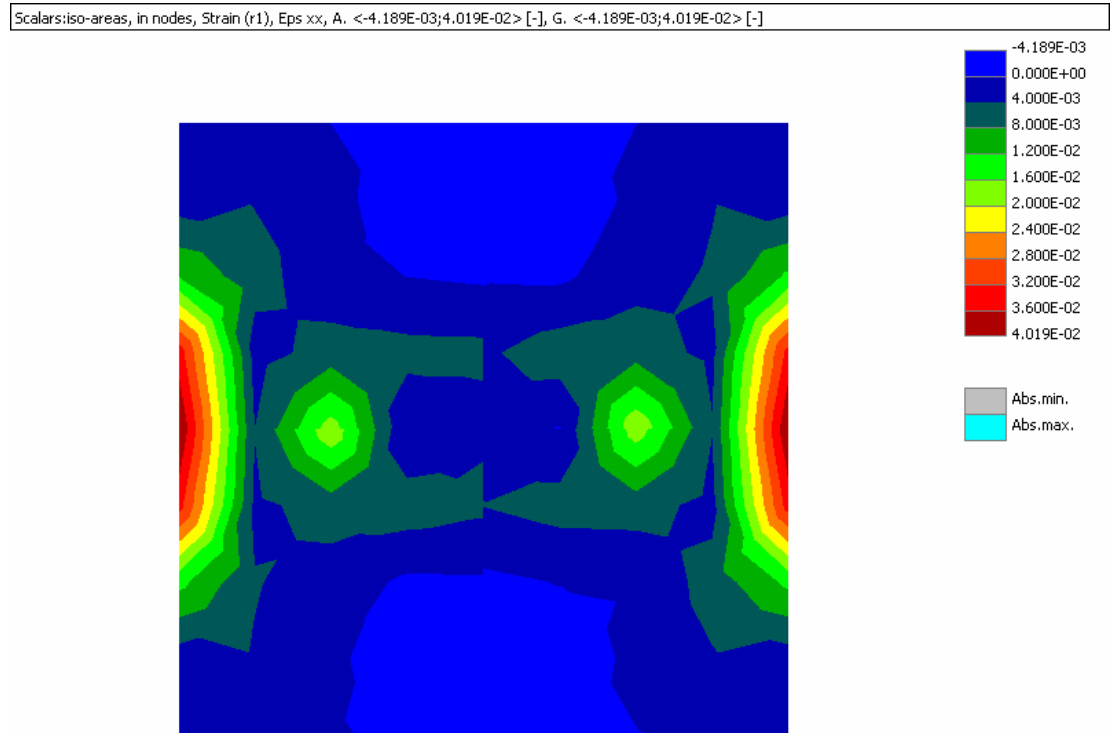
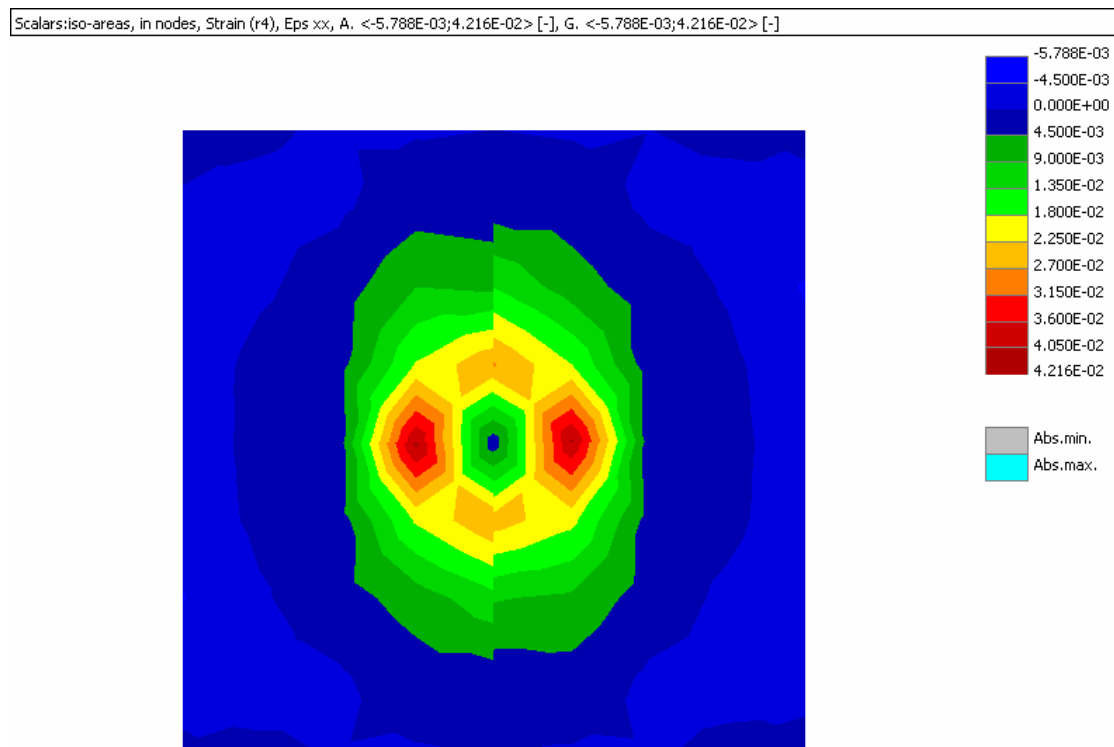


Fig. 4-22 Stresses in reinf. bars near bottom surface (Y - in vertical dir.).



**Fig. 4-23 Strains in reinf. bars near top surface (bars in horizontal dir.).**



**Fig. 4-24 Strains in reinf. bars near bottom surface (bars in horizontal dir.).**

Maximal strains in reinforcing bars are about 4%, which is within the range of tensile capacity.