Probabilistic Estimate of Global Safety Factor – Comparison of Safety Formats for Design based on Non-linear Analysis

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V. Červenka, J. Červenka

Cervenka Consulting, Predvoje 22, 162 00, Praha 6, Czech Republic, www.cervenka.cz

Abstract

The paper proposes a new safety format suitable for design of reinforced concrete structures using non-linear analysis. The safety format is based on global assessment of structural resistance. It is compared with other available safety formats using several structural examples.

1 Introduction

In recent years more engineers use non-linear analysis while designing complex buildings, dams, or bridges. This evolution is supported by rapid increase of computational power as well as by new capabilities of the available tools for numerical simulation of structural performance.

The code provisions hand provide very little guidance how to use the results of a non-linear analysis for structural design or assessment. The safety formats and rules that are usually employed in the codes are tailored for classical design procedures based on hand calculation or linear analysis and local section checks. On the other hand, non-linear analysis is by its nature always a global type of assessment, in which all structural parts, or sections, interact. Until recently the codes did not allow to apply the method of partial safety factors for non-linear analysis, and therefore, a new safety format was expected to be formulated. Certain national or international codes have already introduced new safety formats based on overall/global safety factors to address this issue. Such codes are, for instance, German standard DIN 1045-1 (1998) or Eurocode 2 EN 1992-2, (2005). This paper will try to compare several possible safety formats suitable for non-linear analysis: partial factor method, format based on EN 1992-2, (2005) and fully probabilistic method. A new alternative safety format is also proposed by authors, which is based on semi-probabilistic estimate of variation coefficient of resistance.

Standard design procedure for civil engineering structures based on partial safety factors usually involves the following steps:

1) Conceptual design with initial dimensioning of structural elements based on estimates and engineering judgment.
2) Linear elastic analysis of the structure considering all possible load combinations. Results are actions in some critical sections, which could be referred as *design actions* and can be written as

$$E_d = \gamma_{S1} S_{n1} + \gamma_{S2} S_{n2} + \ldots\ldots \gamma_{Sn} S_{nn} \tag{1}$$

They include safety provisions in which the nominal loads S_{ni} are amplified by appropriate partial safety factors for loading γ_{Si}, where index i stands for load type, and their combinations.

3) *Design resistance* of a section is calculated using design values of material parameters as:

$$R_d = r(f_d, \ldots),\ f_d = f_k / \gamma_m \tag{2}$$

The safety provision of resistance is employed on the material level. Design value of material property f_d is obtained from the characteristic value f_k by its reduction with the partial safety factor γ_m.

4) Safety check of limit state is performed by *design condition*, which requires, that design resistance is greater then design action:

$$E_d < R_d \tag{3}$$

Note, that in the partial safety factor method the safety of material criteria in local points is ensured. However, the probability of failure, ie. probability of violation of the design criteria (3) is not known.

Required reinforcement is designed using steps 2), 3) and 4), and changes in dimensions may be needed. The whole procedure is repeated until all sections satisfy the design criteria that are usually specified by national or international design codes. The final steps of the design verification process often involve assessment of serviceability conditions, i.e. deflections, crack width, fatigue, etc. In certain cases, these serviceability conditions might be the most important factors affecting the final design.

In the above outlined design procedure, the non-linear analysis should be applied in step 2) to replace the linear analysis. Following the current practice designer will continue to steps 3), 4) and perform the section check using the internal forces calculated by the non-linear analysis. This is a questionable practice due to the following reasons. If design values for material parameters are used in the non-linear analysis, then very unrealistic, i.e. degraded, material is assumed. In statically indeterminate structures this may result in quite unrealistic redistribution of forces, which may not be on the conservative side. Furthermore, since in non-linear analysis material criteria are satisfied implicitly within constitutive laws, it does not make sense to continue to step 3) and perform section checks. Instead, a global check of safety should be performed on higher level and not in local sections. This is the reason for an introduction of new safety format for non-linear analysis.

Another issue is that non-linear analysis is becomes useful when it is difficult to clearly identify the sections to be checked. This occurs in structures with complicated geometrical forms with opening, special reinforcement detailing, etc. In such cases, usual models for beams and columns are not appropriate, and non-linear analysis is a powerful alternative.

The above discussion shows that it would be advantageous to check the global structural resistance to prescribed actions rather than checking each individual section and that the safety format based on global assessment is more suitable for design approaches based on non-linear analysis. This approach can bring the following advantages:

(a) The nonlinear analysis checks automatically all locations and not just those selected at critical sections.

(b) The global safety format gives information about the structural safety and redundancy. This information is not available in the classical approach of section verification.

(c) The safety assessment on global level can bring, on one hand, more economic solution by exploiting reserves due to more comprehensive design model, on the other hand, the risk of unsafe design is reduced.

However, the above enthusiastic statements should be accepted with caution. There are many aspects of design, which require engineering judgment. Also many empirical criteria must be met as required by codes. Therefore, a global safety assessment based on non-linear analysis should be considered as an additional advanced design tool, which should be used, when standard simple models are not sufficient.

Non-linear analysis offers an additional insight into the structural behavior, and allows engineers to better understand their structures. On the other hand, non-linear analysis is almost always more demanding then a linear analysis, therefore an engineer should be aware of its limits as well as benefits. Other disadvantage is that the force super-position is not valid anymore. The consequence is that separate non-linear analysis is necessary for each combination of actions.

Finally, a note to terminology will be made. The term for *global* resistance (*global* safety) is used here for assessment of structural response on higher structural level then a cross section. In technical literature, the same meaning is sometimes denoted by the term *overall*. The term *global* is introduced in order to distinguish the newly introduced check of safety on global level, as compared to local safety check in the partial safety factor method. This terminology has its probabilistic consequences as will be shown further in the paper. The proposed global approach makes possible the reliability assessment of resistance, which is based on more rational probabilistic approach as compared to partial safety factors.

2 Safety formats for non-linear analysis

2.1 Design variable of resistance

Our aim is to extend the existing safety format of partial factors and make it compatible with nonlinear analysis. First we introduce new design variable of resistance $R=\boldsymbol{r}(f, a, ..., S)$. Resistance represents a limit state. In a simple case this can be a single variable, such as loading force, or intensity of a distributed load. In general this can represent a set of actions including their loading history. We want to evaluate the reliability of resistance, which is effected by random variation of basic variables f - material parameters, a – dimensions, and possibly others.

The resistance is determined for a certain loading pattern, which is here introduced by symbol of actions S. It is understood that unlike material parameters and dimensions, which enter the limit state function $\boldsymbol{r}$ as basic variables, the loading is scalable, and includes load type, location, load combination and history. It is the objective of the resistance R to determine the loading magnitude for given loading model.

Random variation of resistance is described by a statistical distribution characterized by following parameters:

R_m mean value of resistance,

R_k characteristic value of resistance, , i.e. 5% kvantile of the resistance

R_d design value of resistance.

The design condition is defined in analogy with partial safety factor method by Eq.(3)

In general, E_d and R_d represent set of actions and the limit state is a point in a multi-dimensional space. It is therefore useful to define a resistance scaling factor k_R, which describes safety factor with respect to the considered set of design actions. In the simplified form, considering one pair of corresponding components it can be described as:

$$k_R = \frac{R}{E_d} \tag{4}$$

Then, the design condition (3) can be rewritten as:

$$\gamma_R < k_R \tag{5}$$

Where γ_R is required global safety factor for resistance. Factor k_R can be used to calculate the relative safety margin for resistance

$$m_R = k_R - 1 \tag{6}$$

The task now remains to determine the design resistance R_d. The following methods will be investigated and compared:

(a) ECOV method, i.e. estimate of coefficient of variation for resistance.

(b) EN 1992-2 method, i.e estimate of R_d using the overall safety factor from Eurocode 2 EN 1992-2.

(c) PF method, i.e. estimate of R_d using the partial factors of safety

(d) Full probabilistic approach. In this case R_d is calculated by a full probabilistic non-linear analysis.

Furthermore, the limit function ***r*** can include some uncertainty in model formulation. However, this effect can be treated separately and shall not be included in the following considerations.

It should be also made clear, that we have separated the uncertainties of loading and resistance (and their random behavior). Our task is reduced to describe the resistance side of design criterion (3).

2.2 ECOV method – estimate of coefficient of variation

This method is newly proposed by the authors. It is based on the idea, that the random distribution of resistance, which is described by the coefficient of variation V_R, can be estimated from mean R_m and characteristic values R_k. The underlying assumption is that random distribution of resistance is according to lognormal distribution, which is typical for structural resistance. In this case, it is possible to express the coefficient of variation as:

$$V_R = \frac{1}{1.65} \ln\left(\frac{R_m}{R_k}\right) \tag{7}$$

Global safety factor γ_R of resistance is then estimated as:

$$\gamma_R = \exp(\alpha_R \beta V_R) \tag{8}$$

where α_R is sensitivity (weight) factor for resistance reliability and β is reliability index. The above procedure enables to formulate the safety of resistance in a rational way, based on the principles of reliability accepted by the codes. Appropriate code provisions can be used to identify these parameters. According to Eurocode 2 EN 1991-1, typical values are $\beta = 4.7$ (one year) and $\alpha_R = 0.8$. In this case, the global resistance factor is:

$$\gamma_R \cong \exp(3.04 V_R) \tag{9}$$

and the design resistance is calculated as:

$$R_d = \frac{R_m}{\gamma_R} \tag{10}$$

The key factor in the proposed method is to determine the mean and characteristic values R_m , R_k. It is proposed to estimate them using two separate nonlinear analyses using mean and characteristic values of input material parameters, respectively.

$$R_m = r(f_m, \ldots)\,, \quad R_k = r(f_k, \ldots) \tag{11}$$

The method is general and reliability level β and distribution type can be changed if required. The advantage of this approach is that the sensitivity to individual parameters such as for instance steel or concrete strength can be estimated. The disadvantage is the need for two separate non-linear analyses.

2.3 EN1992-2 method

Design resistance is calculated from

$$R_d = r(\tilde{f}_{ym}, \tilde{f}_{cm} \ldots, S) / \gamma_R \tag{12}$$

Material properties used for resistance function are used as follows:

Table 1: Material parameters used in EN1992-2 method

$\tilde{f}_{ym} = 1.1 f_{yk}$	Steel yield strength
$\tilde{f}_{pm} = 1.1 f_{pk}$	Prestressing steel yield strength
$\tilde{f}_{cm} = 1.1 \frac{\gamma_s}{\gamma_c} f_{ck}$	Concrete compressive strength, where γ_s and γ_c are partial safety factors for steel and concrete respectively. Typically this means that the concrete compressive strength should be calculated as $\tilde{f}_{cm} = 0.843 f_{ck}$

The global factor of resistance shall be $\gamma_R = 1,27$

The evaluation of resistance function is done by nonlinear analysis assuming the material parameters according to the above rules.

2.4 PSF method – partial safety factor estimate

Design resistance R_d can be estimated using design material values as

$$R_d = r(f_d, ..., S) \tag{13}$$

In this case, the structural analysis is based on extremely low material parameters in all locations. This may cause deviations in structural response, e.g. in failure mode. It may be used as an estimate in absence of a more refined solution.

2.5 Full probabilistic analysis

Probabilistic analysis is a general tool for safety assessment of reinforced concrete structures, and thus it can be applied also in case of non-linear analysis. A limit state function can be evaluated by means of numerical simulation. In this approach the resistance function $r(\mathbf{r})$ is represented by non-linear structural analysis and loading function $s(\mathbf{s})$ is represented by action model. Safety can be evaluated with the help of reliability index β, or alternatively by failure probability P_f taking into account all uncertainties due to random variation of material properties, dimensions, loading, and other.

Probabilistic analysis based on numerical simulation include following steps:

(1) Numerical model based on non-linear finite element analysis. This model describes the resistance function $r(\mathbf{r})$ and can perform deterministic analysis of resistance for a given set of input variables.

(2) Randomization of input variables (material properties, dimensions, boundary conditions, etc.). This can also include some effects of actions, which are not in the action function $s(\mathbf{s})$ (for example pre-stressing, dead load etc.). Random properties are defined by random distribution type and its parameters (mean, standard deviation, etc.). They describe the uncertainties due to statistical variation of resistance properties.

(3) Probabilistic analysis of resistance and action. This can be performed by numerical method of Monte Carlo-type of sampling, such as LHS sampling method. Results of this analysis provide random parameters of resistance and actions, such as mean, standard deviation, etc. and the type of distribution function for resistance.

(4) Evaluation of safety using reliability index β or probability of failure.

Probabilistic analysis can be also used for determination of design value of resistance function r ($\mathbf{r}$) expressed as R_d. Such analysis involves the steps (1) to (3) above and R_d is determined for required reliability β or failure probability P_f .

2.6 Non-linear analysis

Examples in this paper are analysed with program ATENA for non-linear analysis of concrete structures. ATENA is capable of a realistic simulation of concrete behavior in the entire loading range with ductile as well as brittle failure modes as shown in pepers by Cervenka (1998), (2002). The numerical analysis is based on the finite element method and non-linear material models for concrete, reinforcement and their interaction. Tensile behavior of concrete is described by smeared cracks, crack band and fracture energy, compressive behavior of concrete is described by damage model with hardening and softening. In the presented examples the reinforcement is modelled by truss elements embeded in two-dimensional isoparametric concrete elements. Nonlinear solution is performed incrementally with equlibrium iterations in each load step.

3 Examples of application

The performance of presented safety formats will be tested on several examples ranging from simple determinate structures with bending failure mode up to statically indeterminate structures with shear failure modes.

3.1 Bending problem, statically determinate structure

The first example is a very simple problem (see Figure 1) with bending failure mode. Its resistance will be analyzed using the presented safety formats, and it will be also compared with the classical cross-section check using the partial factor method.

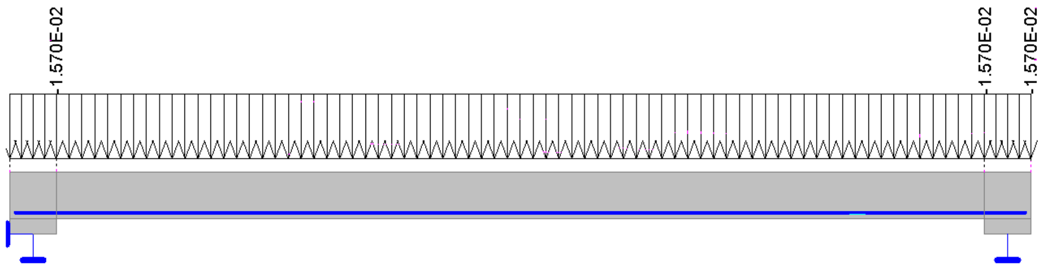

Figure 1: Beam geometry for safety formats comparison and distributed design load.

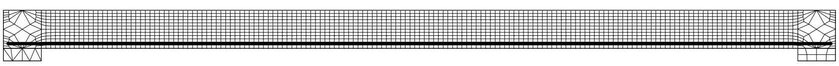

Figure 2: Finite element meshes used in the bending example

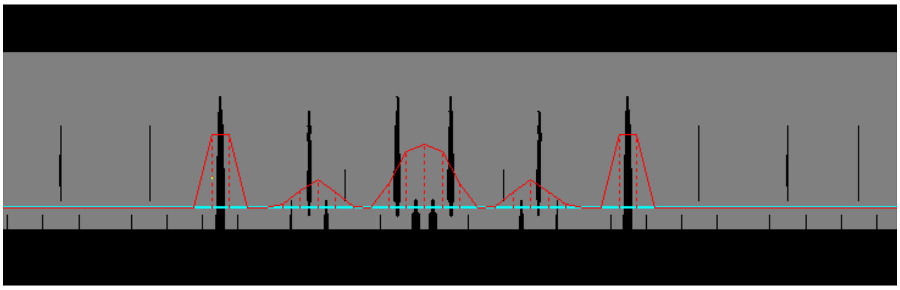

Figure 3: Typical result from finite element analysis using the program ATENA (Cervenka Consulting) showing crack pattern and reinforcement stress.

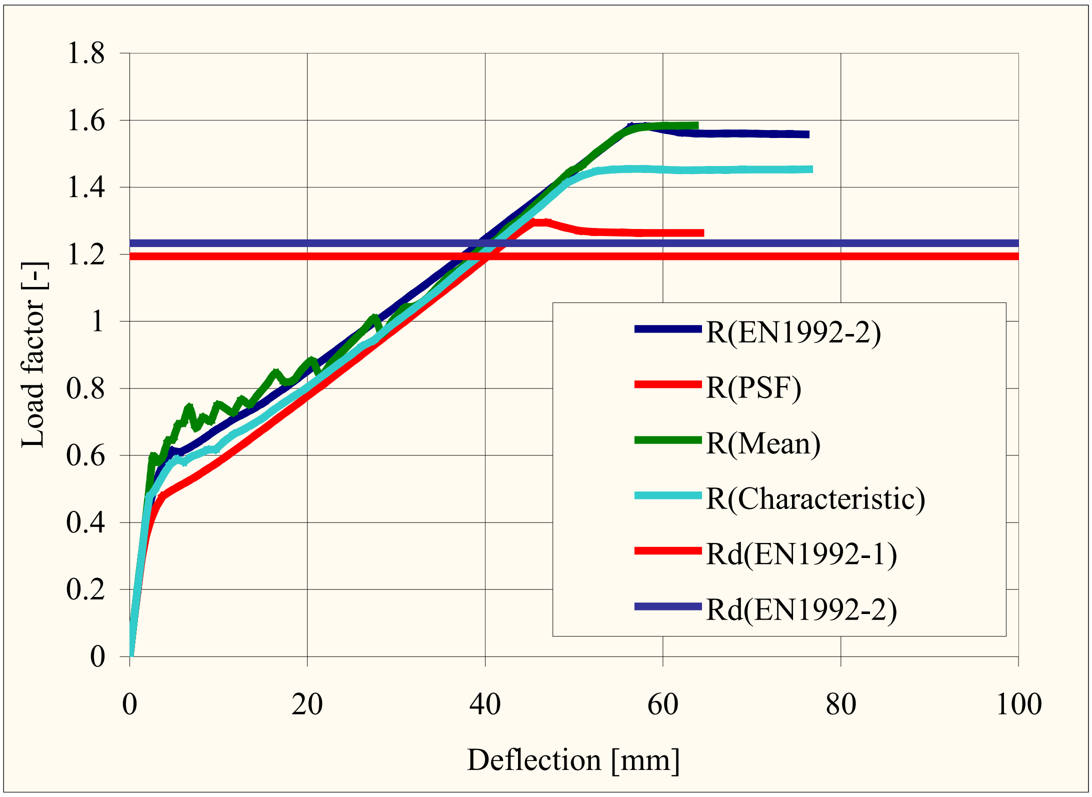

Figure 4: Comparison of load-displacement curves for different safety formats for the bending problem

Figure 4 contains the comparison of response curves as well as design resistance values by the different safety approaches. The numerical values of design resistance are for clarity also summarized in Table 2. The table demonstrates that for this simple case that can be easily checked by hand calculation all the methods give identical results. This is to be expected since the advantages of the more advanced methods will appear namely in cases when hand calculation and standard approaches are not applicable or introduce large simplifications. Such simplifications usually lead to significant underestimation of design resistance values.

Table 2: Comparison of calculated values for design resistance using various safety formats

Safety Format	Scaling factor k_R
Standard partial factor based on EN1992-1	1.19
ECOV method	1.29
EN1992-2 method	1.23
PSF method	1.29
Full probabilistic approach	1.21

3.2 Shear problem, statically indeterminate structure

The objective of the example presented in this section is to compare the results obtained by various safety format approaches on a more complicated problem of statically indeterminate structure with shear failure. The analyzed example has been tested experimentally by Asin (1999), as shown in Figure 5. Thus, it is also possible to check the analytical results with an experimental behavior.

The beam geometry with dimensions and material properties is shown in Figure 6 and the subsequent Figure 7 depicts the used numerical model and boundary conditions.

Several safety formats, as they are described in Section 2, are used to analyzed this shear beam. The used methods and the necessary material properties are listed in Table 1.

Figure 5: Shear wall tested in the laboratory, Asin (1999)

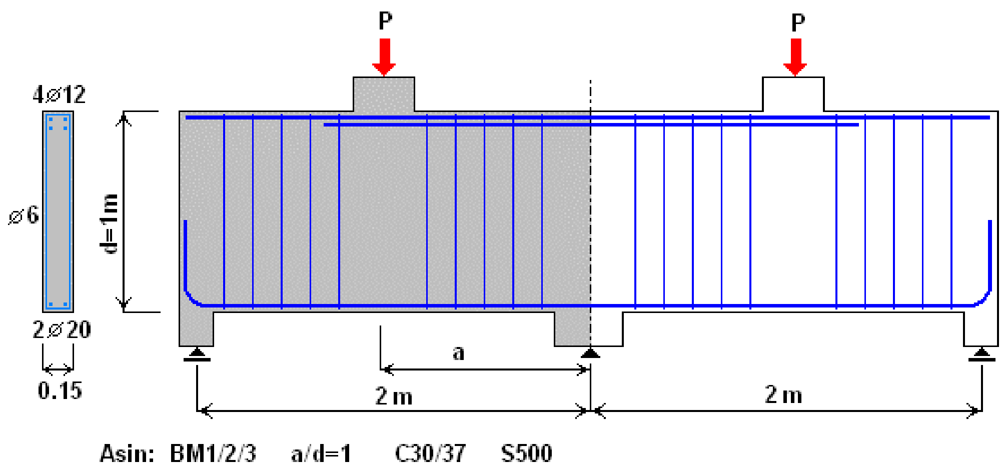

Figure 6: Geometry of the shear wall example

Table 3: Material properties used for various safety formats

Safety format	Partial (PSF) EN1992-1	Global EN1992-2	Probabilistic mean	st.dev.	Dist.
Concrete 30/37					
E_c GPa	*32*	*32*	*32*	*4.1*	*lognrm.*
f_c MPa	*20*	*25*	*38*	*4.9*	*lognrm.*
f_t MPa	*1.3*	*1.7*	*2.9*	*0.6*	*Weibull*
G_f N/m	*35*	*44*	*66*	*13*	*Weibull*
Steel 500					
f_{sy} MPa	*434*	*550*	*550*	*31*	*lognrm.*

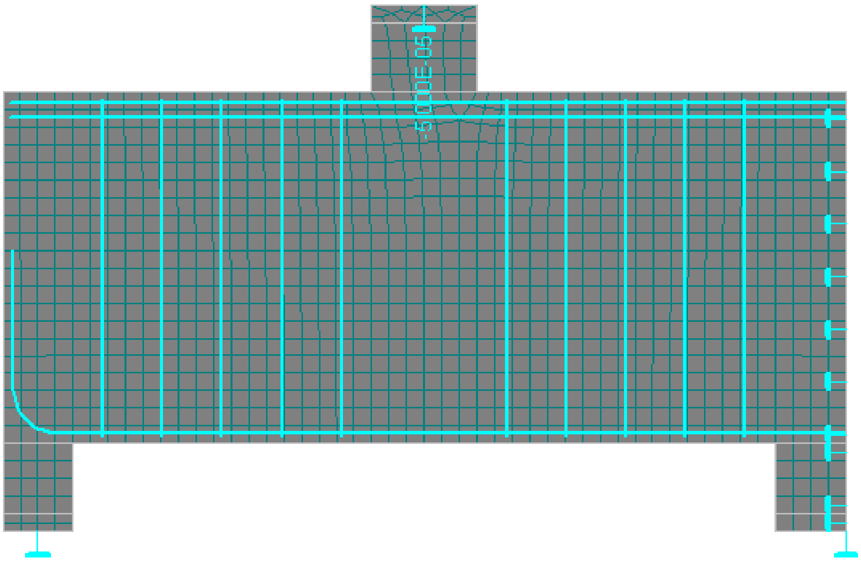

Figure 7: Finite element model for shear wall example

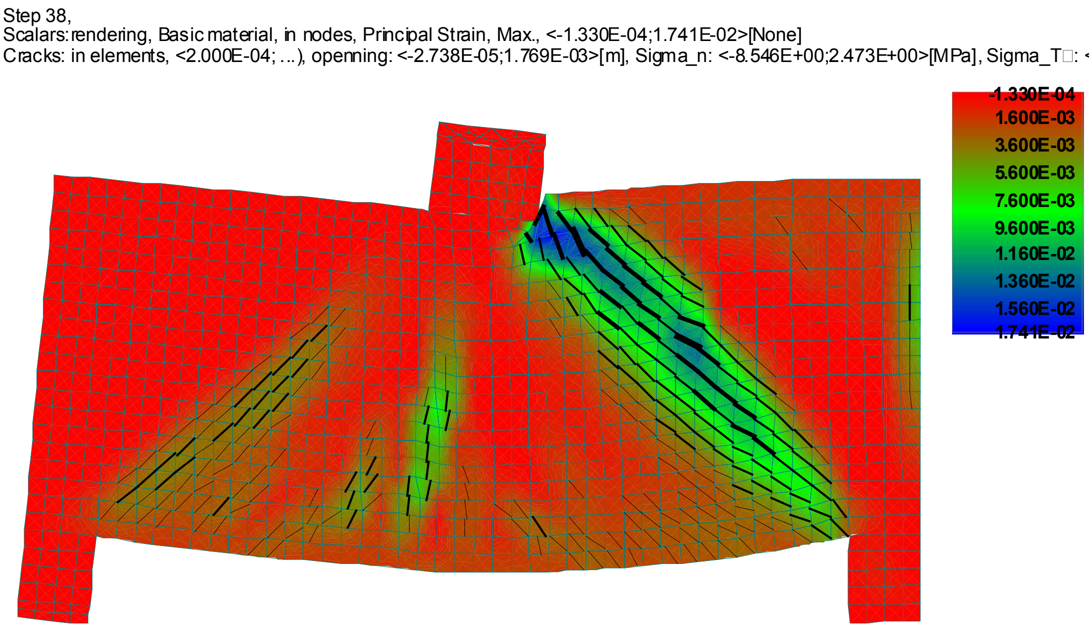

Figure 8: Final failure mode calculated by nonlinear analysis

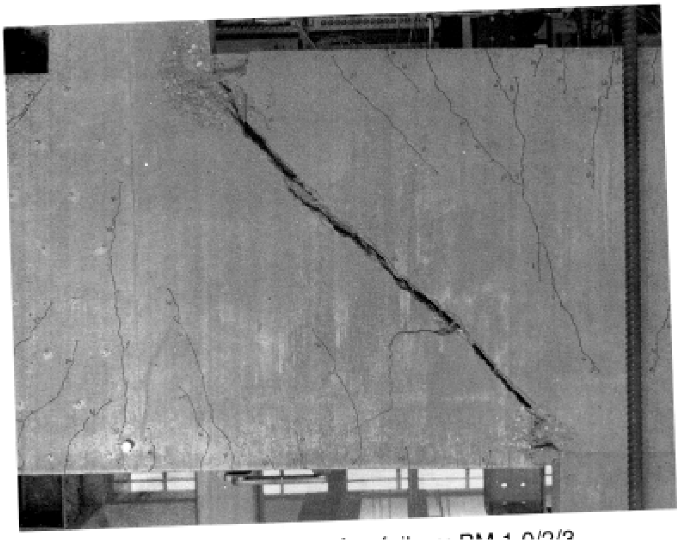

Figure 9: Typical result from laboratory experiment

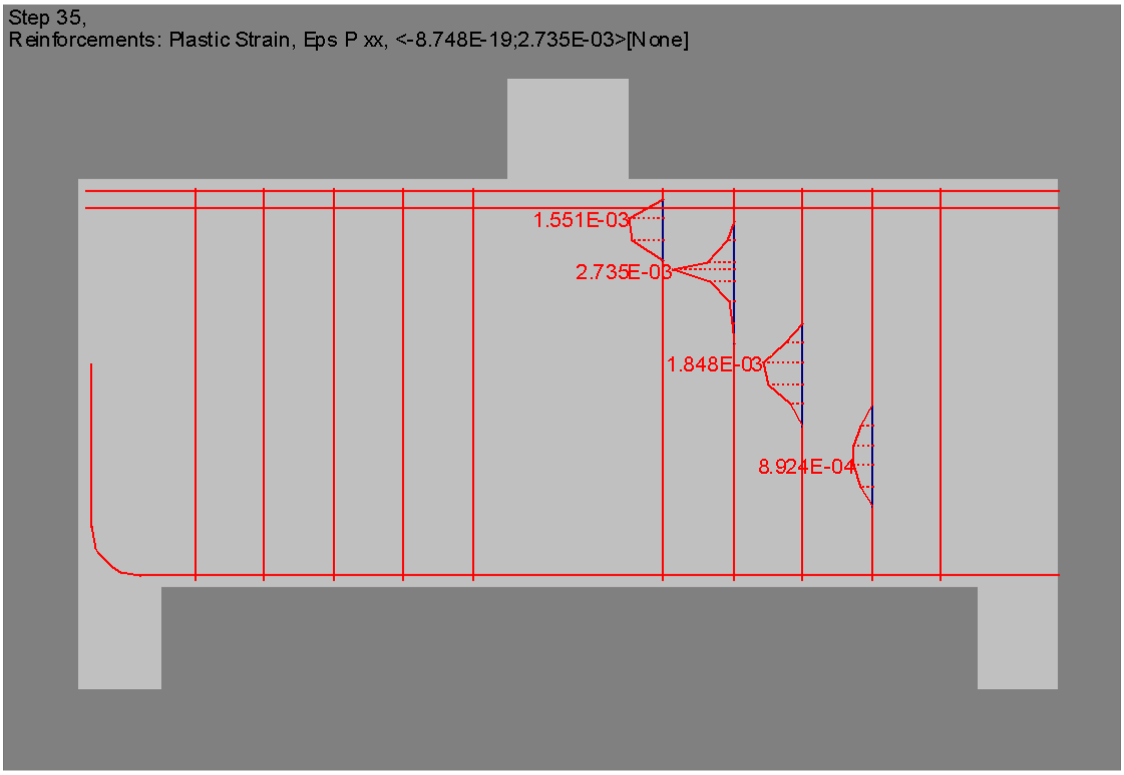

Figure 10: Reinforcement yielding at failure from nonlinear analysis

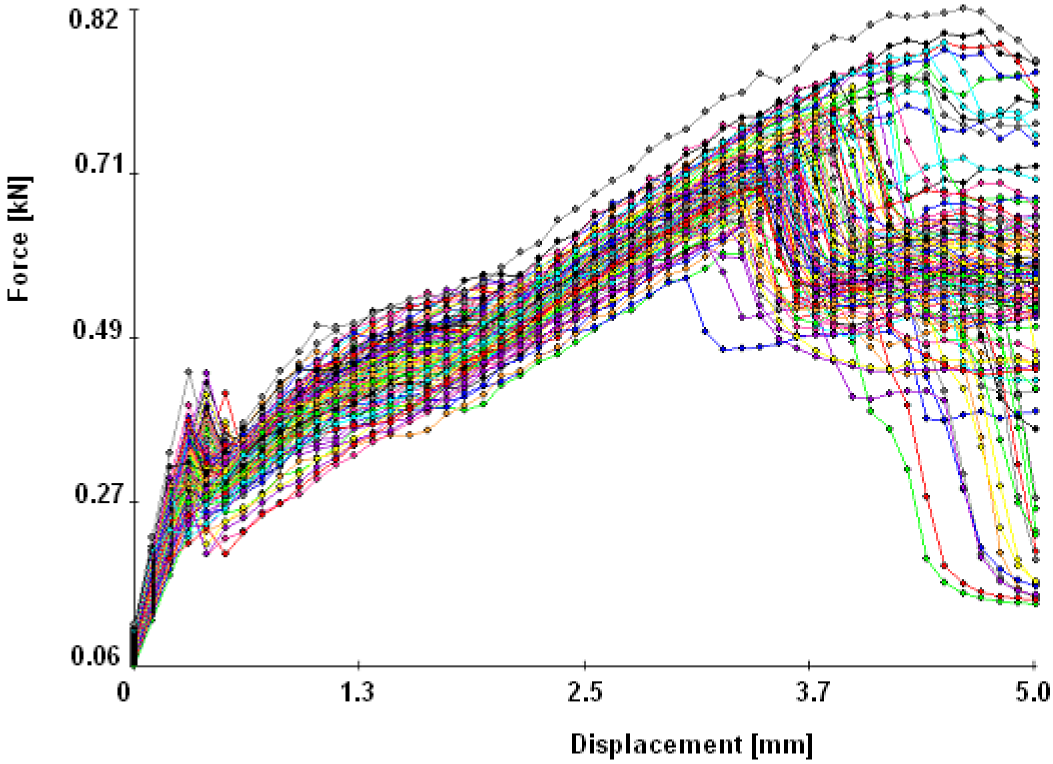

Figure 11: Set of load-displacement curves calculated by full probabilistic nonlinear analysis

Various safety formats are used to determine the design resistance of the analyzed structure. In this case the solution given by partial safety factors (PSF) is taken as a reference. Thus, global factors are not evaluated and design resistance R_d is directly compared for various approaches in Table 4. It shows that again all the methods give similar resistance values. The only difference is in the case of probabilistic methods where a strong sensitivity with respected to the selected shape of distribution function. In case of very realistic lognormal distribution, the calculated design resistance 512 is quite similar to those calculated by PSF and global resistance method. For other statistical distribution functions the calculated value could be either lower or higher. This demonstrates the importance of sufficient knowledge of statistical properties of input parameters in we want to rely on probabilistic analysis.

Table 4: Comparison of calculated design resistances by various safety methods

Method	Design force R_d [kN]	R_d / R_d^{PSF}
PSF	501	1.0
EN 1992-2	490	0.98
Full probabilistic using normal distrib. for $r(f,...)$	512	1.02
Full probabilistic using Weibul distrib. for $r(f,...)$	620	1.24
Full probabilistic using all normal materials and $r(f,...)$	465	0.93

4 Conclusions

The paper presents new safety format that is suitable for reinforced concrete design based on non-linear analysis. The new method is called ECOV (Estimate of Coefficient Of Variation). The advantage of the proposed method is that it can capture the resistance sensitivity to the random variation of input variables, and thus it can reflect the effect of failure mode on safety. It requires two nonlinear analyses with mean and characteristic values of input parameters respectively. Other safety formats suitable for non-linear analysis that are based on global resistance are presented. They are: the approach proposed by EN 1992-2, fully probabilistic analysis and a simple approach based on design values of input parameters, i.e. characteristic parameters reduced by partial safety factors. The last approach is usually not recommended by design codes, but practicing engineers often overlook this fact, and use this approach if a non-linear analysis is available in their analysis tools. The consequences are investigated in this paper.

The discussed safety formats are tested on two examples. First one is a simple supported beam in bending, and the second one is a statically indeterminate shear wall. For the investigated range of problems, which is quite narrow but still representative, all the methods provide quite reliable and consistent results.

Based on the limited set of examples the following conclusions are drawn:

- The proposed EVC method gives consistent results compare to other approaches.
- The PSF method, which uses input parameters with partial safety factors appears to be sufficiently reliable and it is a natural extension of the classical approach to the modern design methods based on non-linear analysis.
- Fully probabilistic analysis is sensitive to the type of random distribution assumed for input variables. It can provide additional load-carrying capacity if statistical properties of the analyzed system are known or can be accurately estimated.

The methods are currently subjected to further validation by authors for other types of structures and failure modes.

5 References

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