

# Fracture-plastic model for the analysis of reinforced concrete structures subjected to fire

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## Summary

This paper summarizes modifications and extensions to the previously published combined “fracture-plastic” material model for modelling of reinforced concrete structures subjected to fire. The behavior of the model is demonstrated on simple uniaxial tests as well as a three-dimensional analysis of a tunnel fire.

**Keywords:** Fracture, plasticity, reinforced concrete, fire, finite element analysis.

## 1. Introduction

Numerous thermal processes occurring in a heated concrete may render the use of the general models too difficult for engineering practice. The main disadvantage of such models is a lack of accurate values for numerous initial material constants or boundary conditions. From experiments only an elasticity modulus, strength in tension and compression or a permeability varying with temperature usually can be obtained. The presented paper describes the modification to fracture-plastic model developed by the authors and firstly presented in [1]. The presented work describes the extension of the model for the analysis of structures subjected to fire, where the material properties of concrete as well as reinforcement are strongly dependent on the temperature.

## 2. Material model for concrete

Concrete is a complex material with strongly non-linear response even under service load conditions.

*Tensile behaviour* of concrete is modelled by non-linear fracture mechanics with simple Rankine based criterion. This approach is combined with the crack band method. In this method the smeared crack concept is accepted. Its parameters are: tensile strength, shape of the stress-crack opening curve and fracture energy, see

Fig. 1, based on ref. [2],[3],[4]. In this formulation the crack strain is related to the element size. Consequently, the softening law in terms of strains for the smeared model is calculated for each element individually, while the crack-opening law is preserved.

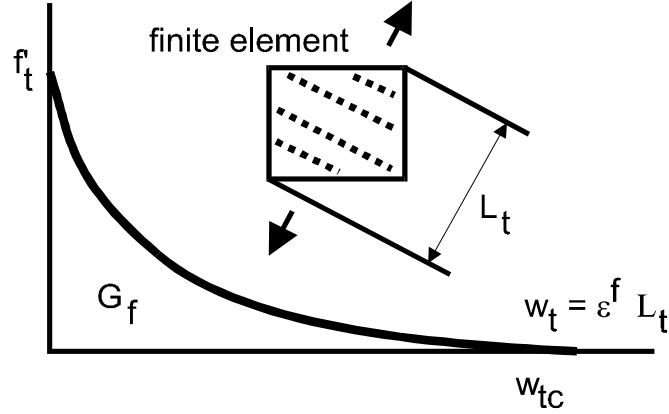


Fig. 1: Tensile softening law with crack band model definition.

*Compressive behavior* is modelled using a plasticity based model with failure surface defined by Menetrey-Willam three-parameter criterion [5] (see Fig. 2).

$$F_{3P}^p = \left[ \sqrt{1.5} \frac{\rho}{f_c'} \right]^2 + m \left[ \frac{\rho}{\sqrt{6} f_c'} r(\theta, e) + \frac{\xi}{\sqrt{3} f_c'} \right] - c = 0 \quad (1)$$

where

$$m = 3 \frac{f_c'^2 - f_t'^2}{f_c' f_t'} \frac{e}{e+1}, \quad r(\theta, e) = \frac{4(1-e^2) \cos^2 \theta + (2e-1)^2}{2(1-e^2) \cos \theta + (2e-1) [4(1-e^2) \cos^2 \theta + 5e^2 - 4e]^{\frac{1}{2}}}$$

In the above equations  $(\xi, \rho, \theta)$  are Heigh-Vestergaard coordinates,  $f_c'$  and  $f_t'$  is compressive strength and tensile strength respectively. Parameter  $e \in (0.5, 1.0)$  defines the roundness of the failure surface. The failure surface has sharp corners if  $e = 0.5$ , and is fully circular around the hydrostatic axis if  $e = 1.0$ . The surface evolves during the yielding/crushing process by the laws denoted in Fig. 3. Special iterative algorithm is developed [1] to solve the plastic and fracture models such that the final stress tensor in both models are identical. This algorithm is schematically shown in two-dimensions in Fig. 4.

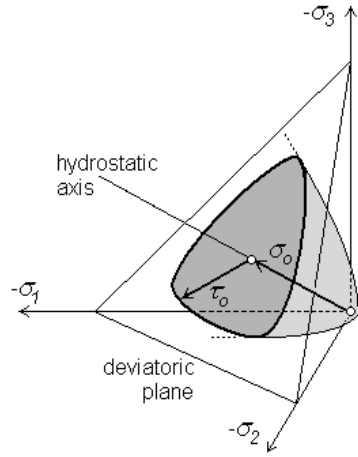


Fig. 2: Shape of the 3D Menetréy-Willam failure criterion [5] for concrete in three-dimensional space of principal stresses.

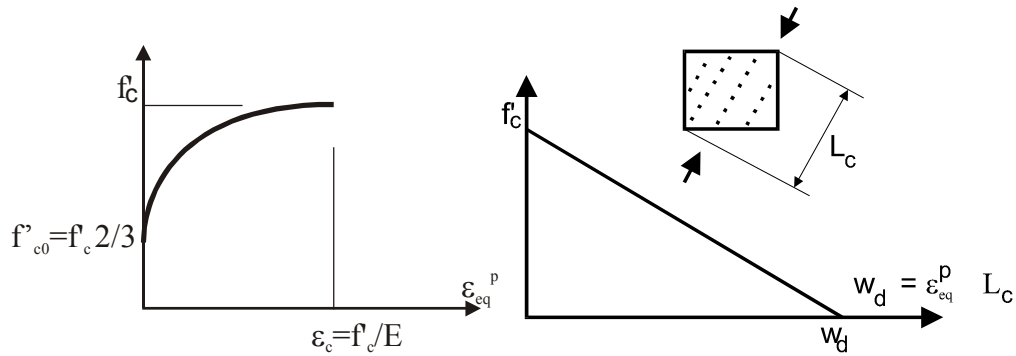


Fig. 3: Compression hardening/softening laws.

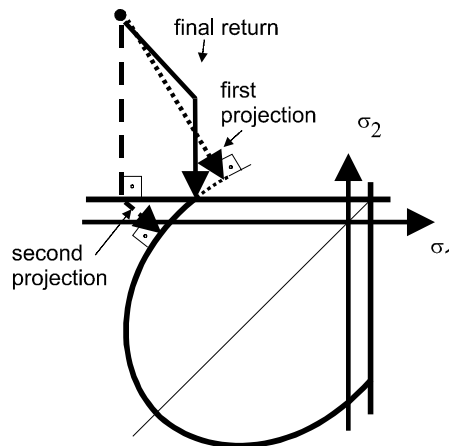


Fig. 4: Schematic description of the algorithm for the combination of fracture-plastic models.

### 3. Material model for reinforcement

Finite element modelling of reinforced concrete structures requires special tools for modelling of all types of reinforcement. Most of these models are illustrated in Fig. 5 for three-dimensional solids.

Concrete is modeled by solid elements. Concrete to concrete, or concrete to other material interfaces can model the frictional type of interaction between structural elements. The mesh type of reinforcement can be represented as smeared reinforcement. In this element, the individual bars are not considered, while reinforcement is considered as a component of the composite material.

Individual bars can be modelled by truss elements embedded in concrete elements with axial stiffness only. In this technique the mesh is generated first for concrete. Then the bar elements are embedded in this mesh. The bar element can be considered as any other element but its nodes are made kinematically dependent on concrete nodes. Thus the reinforcing is not affecting the mesh generation. The described family of finite elements make it possible to cover most practical cases of reinforced concrete structures.

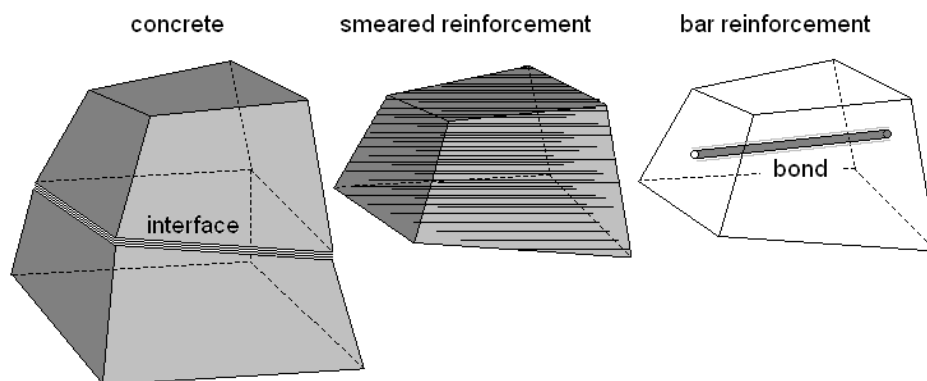


Fig. 5: Three-dimensional elements of reinforced concrete

### 4. Temperature dependent material properties

The used material models for concrete as well as reinforcement are formulated in a purely incremental manner, and the selected material parameters are temperature dependent. The temperature dependent evolution laws of these parameters are shown in Fig. 6. They have been derived based on Eurocode formulas [6] and the experimental results [7]. Analogical dependence is used also for the stress-strain law for reinforcement, where this law is scaled based on the maximally reached temperature at each reinforcement element based on the Eurocode formulas for reinforcement yield strength temperature dependence [6].

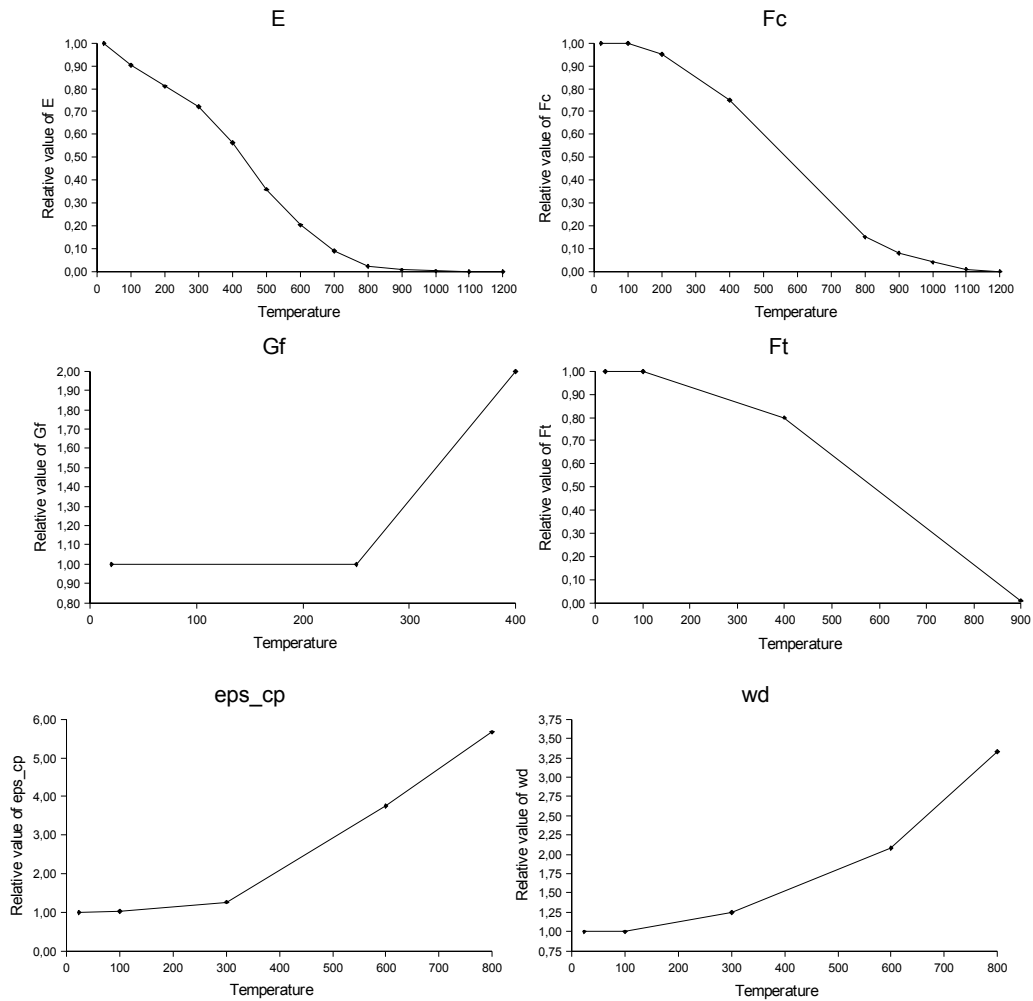


Fig. 6: Evolution laws for temperature dependence parameters.

## 5. Examples

Several examples are presented to demonstrate the behavior of the proposed model in simple uniaxial tests as well as on a more complicated example of a tunnel suspended ceiling subjected to fire.

### 5.1 Uniaxial tests

The tests consisted of standard compression and tension test simulation, where the specimen was loaded to collapse during uniform temperature field distribution. The temperatures chosen were in range 100°C to 800 °C with a step of 100 °C. The results show the behaviour of material model during different temperatures and are depicted in Fig. 7 and Fig. 8

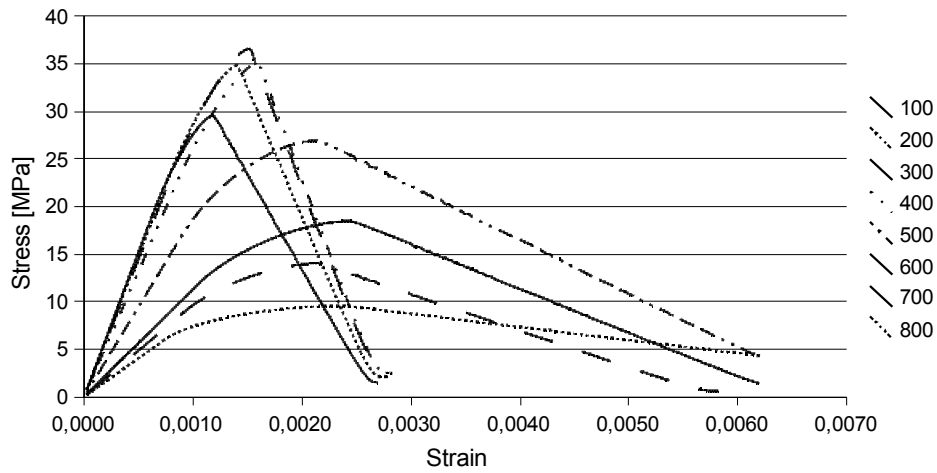


Fig. 7: Uniaxial compression test at different temperatures.

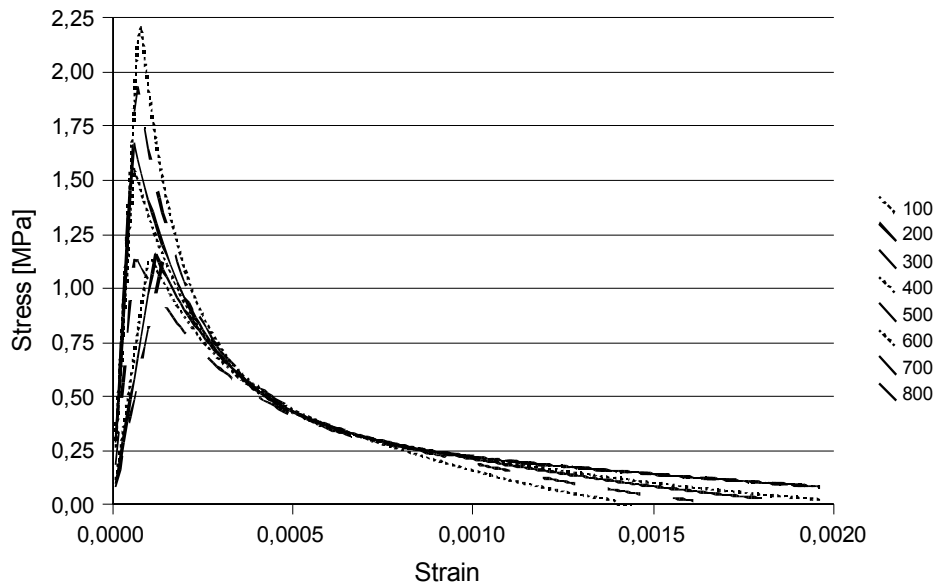


Fig. 8: Uniaxial tension test at different temperatures.

## 5.2 Suspended ceiling simulation

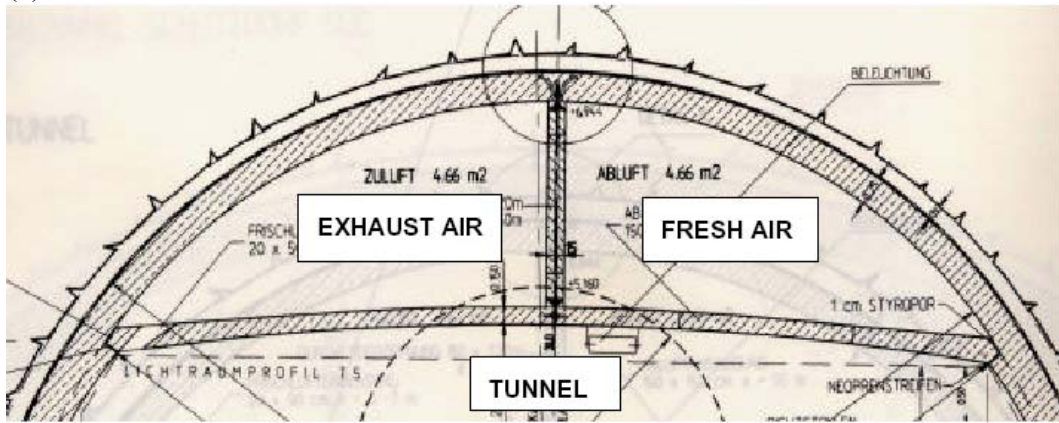
The structure consists of a reinforced concrete suspended ceiling slab with total span of 8.0 m, suspended in the mid-span by pair of hangers. The geometry corresponds to a specimen, which will be tested during a european research project UPTUN. The presented results are part of a pre-test simulation program. At both ends the slab is simply supported with possibility of rising. The thickness of the slab is 200 mm, width is 2.0 m.

The symmetry of structure was employed in the analysis, so the model (Fig. 9) consists of one quarter of the structure.

The structure was exposed with bottom size to Modified HC fire scenario. For the upper surface condition the constant ambient temperature of 20 °C. No thermal shielding was considered. The structure was initially loaded with assumed self weight of 5 kN/m<sup>2</sup> and live load of 3.5 kN/m<sup>2</sup>.

First transient thermal analysis was performed with temperature dependent material properties was performed the obtained thermal fields were used in subsequent incremental stress analysis, in which the models described in Sections 2,3, and 4 were utilized.

(a)



(b)

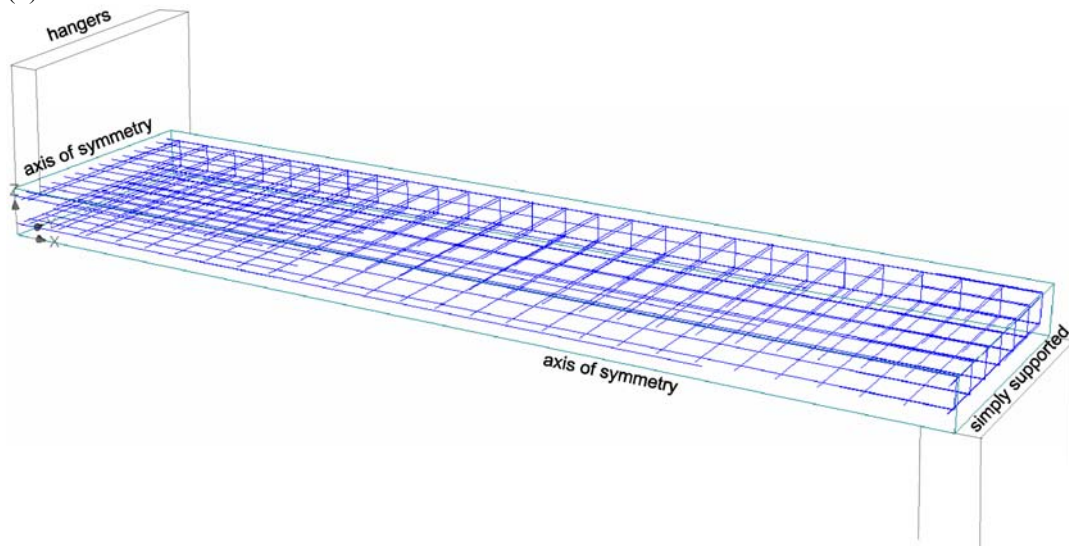


Fig. 9: (a) Typical tunnel design with suspended ceiling (figure taken from UPTUN report) (b) Geometrical model for the suspended ceiling simulation.

The deformed shape and calculated crack pattern after 3 hours of modified hydrocarbon fire is shown in Fig. 10. The structure suffered an extensive damaged, but full collapse was not reached in the investigated time period.

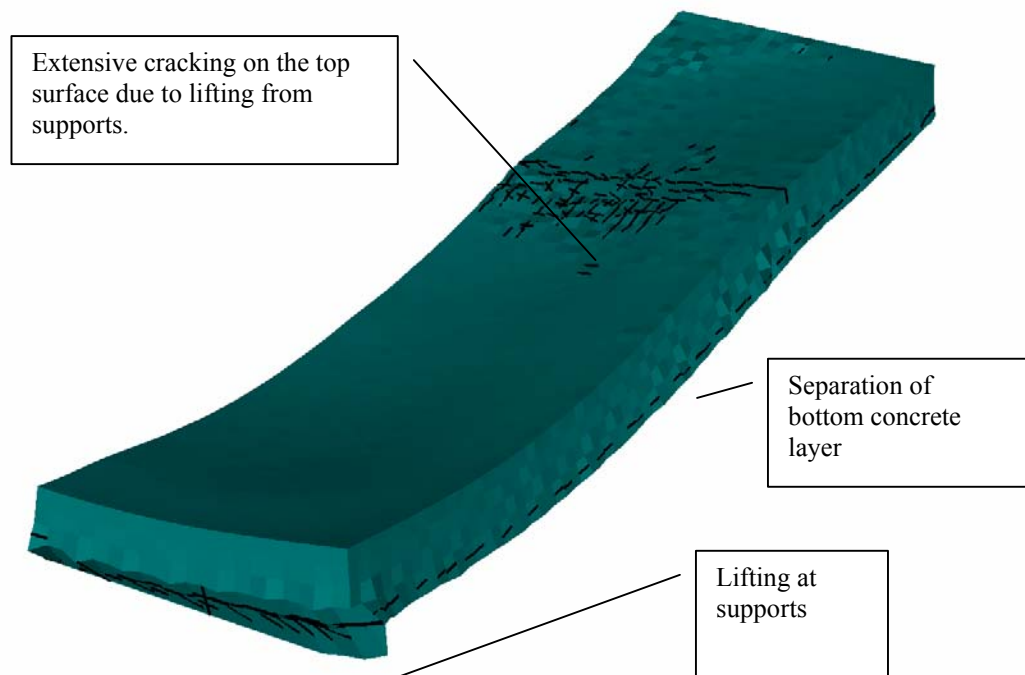


Fig. 10: Deformed shape and crack pattern after 3 hours of modified HCM fire.

## 6. Discussion, Conclusions and Acknowledgements

The paper describes modification to combined fracture-plastic model [1] to extend its applicability for the simulation of reinforced concrete structures subjected to fire. Several examples are presented ranging from simple uniaxial tests up to more complicated problem of a full three-dimensional analysis of a tunnel suspended ceiling. The presented work is part of a european research project UPTUN GRID-CT-2002-00766. The financial support from the european community is greatly appreciated.

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## 8. Authors' Data

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