

# On the choice between discrete or smeared approach in practical structural FE analyses of concrete structures

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**Abstract.** The paper concerns with methods of modelling crack behaviour in quasi-brittle materials like concrete and rocks. It discusses two ultimate approaches, namely smeared crack approach on one side and discrete crack linear and non-linear method on the other side of the spectrum of techniques available to solve the problem. Pros and cons of the presented material models are identified and some hints for their practical use are given. Although the discrete crack approach is sometimes preferred for being based on a mature theory, the smeared crack analysis seems to be more often applicable for usual R/C analyses from engineering practice. This is documented on a few sample analyses of concrete and reinforced concrete structures. Finally, a pilot example is presented combining the both approaches in a single analysis.

## 1. Introduction

A proper way of modelling material behaviour is one of fundamentals that is essential for realistic analyses of concrete structures. The description of cracking and failure within finite element analysis of quasi-brittle structures and materials such as concrete has led to two fundamentally different approaches: the discrete and the smeared one. They represent two distinct viewpoints on the problem of modelling damage in quasi-brittle materials.

Smeared model that was first introduced by Rashid (1968) and Červenka & Gerstle (1971,1972) is based on the development of appropriate continuum material models, in which cracks are smeared over a distinct area, typically finite element or an area corresponding to an integration point of the finite element. It builds up on an equivalent continuum concepts of elastic degradation and/or softening plasticity within the fixed mesh approach. Although a few commercial packages support nowadays a discrete crack model as well (at least in a simplified manner), smeared model seems to retain its dominant role for engineering practitioners. Based on the authors' experience, this approach is adequate for most practical analyses, and it delivers reasonably accurate results at a lower cost than is needed by discrete models.

In the discrete approach, that was first introduced to concrete structures by Saouma & Ingraffea (1981), the discontinuities of the displacement field resulting from the failure process are introduced directly into the numerical model. The discrete approach is directly based on the principles of fracture mechanics or the fictitious crack concepts. This method is theoretically more suitable to capture the failure localisation. On the other hand, an adaptive re-meshing technique is required to account for phenomena such as progressive failure. This and other requirements means that discrete models are usually more demanding in the sense that they need more specialised software. It is probably the main disadvantage of the discrete crack concept. It requires the development of a rather complex software package, in which the finite element model is tightly coupled with the geometrical one.

In recent years, new solution methods emerged, such as mesh adaptation in smeared calculations, generalisations of the fictitious crack method with zero and finite thickness interfaces, as well as continuum elements with embedded cracks that blur the well-defined separation of a crack and the cracked element. The latter technique developed e.g. in Jirásek (2000) is of particular interest. In this approach crack is represented either by cracking induced deformations by inelastic strain or by its direct representation of discontinuities in the displacement field within a finite element. This technique was designed to take advantages of both discrete and smeared cracks methods. However, it is still in a stage of development and thus its practical use remains problematic. This paper focuses on methods already available for engineering practice and hence this “hybrid” method is not discussed in more details.

## 2. Smeared crack models

It is beyond the scope of this paper to describe all available smeared crack models. Therefore, the main features and behaviour will be documented on a typical representative, namely the smeared crack model developed by Červenka V. (1985). This model was first implemented in a finite element program Sbeta (1995), and it will be referred to as the Sbeta material model. The examples in this paper are calculated with the new version of the model that is implemented in the program Atena (2000), a new and advanced version of the program Sbeta. This model represents a classical type of the smeared crack formulation. After crack initiation that is controlled by a bi-axial failure envelope (see Fig. 1), the originally isotropic material formulation changes to an orthotropic one. The unloading moduli for each direction are determined from an equivalent uniaxial diagram shown in Fig. 1. For details about the model formulation the reader is referred to the original papers Červenka V. (1985) or Červenka and Pukl (1992).

The smeared crack models are usually formulated in a stress-strain space. In the case of softening materials, this can lead to strong mesh size dependence and zero energy dissipation in the finite element calculations. Therefore, special provisions are necessary to introduce some of the energy concepts of fracture mechanics. In the Sbeta material model this is accomplished using a variant of the crack band model that was originally proposed by Bažant and Oh (1983).

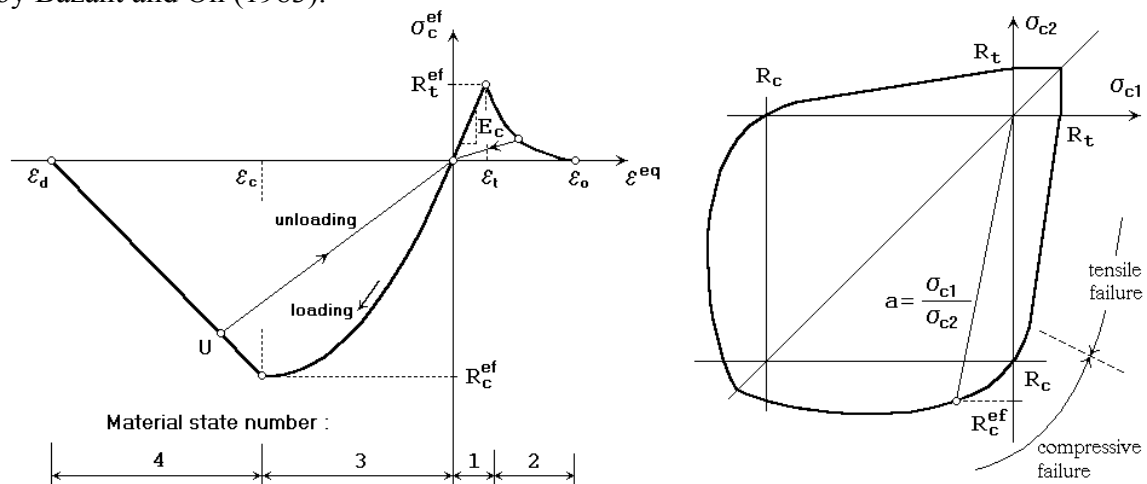


Fig. 1: The equivalent uni-axial stress-strain relationship for the Sbeta material model and the bi-axial failure envelope.

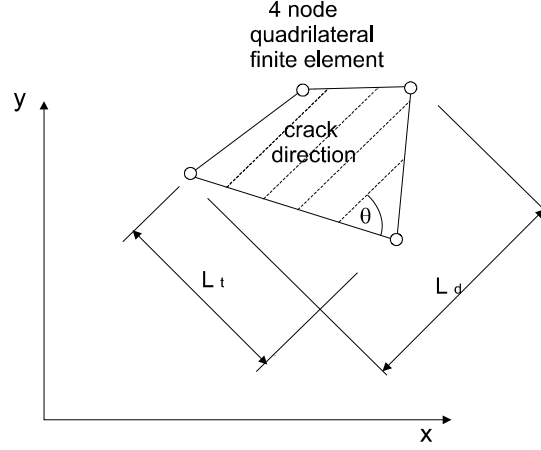


Fig. 2: The determination of crack band sizes for tension and compression.

The stress on softening part of the stress-strain curve is determined based on the crack opening displacement  $w$ , which is calculated from the inelastic cracking strains  $\varepsilon_{cr}$  using the following formula:

$$w = \varepsilon_{cr} \gamma L_t \quad (1)$$

where  $L_t$  is the crack band size that is determined from the element size projected into the direction perpendicular to the crack (see Fig. 2), and  $\gamma$  is a factor depending on the crack direction with respect to the element side orientation (Červenka V. et. al. 1995).

The softening stress is then calculated using a particular softening relationship. There are various softening formulas in the Sbeta model, but the formula proposed by Hordijk (1991) is used in the examples throughout this paper:

$$\frac{\sigma}{R_t} = \left\{ 1 + \left( c_1 \frac{w}{w_c} \right)^3 \right\} \exp\left( -c_2 \frac{w}{w_c} \right) - \frac{w}{w_c} (1 + c_1^3) \exp(-c_2), \quad \text{where } w_c = 5.14 \frac{G_F}{R_t} \quad (2)$$

where  $R_t$  is the tensile strength,  $c_1, c_2$  are constants equal to 3 and 6.93 respectively,  $w_c$  the critical tensile displacement and  $G_F$  is the fracture energy.

A similar approach is adopted also in compression, where the softening is assumed to be linear and controlled by the critical compressive displacement  $w_d$  using the crush band size  $L_d$ . This formulation is based on the experimental observations by van Mier (1986).

### 3. Discrete cracks models based on linear and nonlinear fracture mechanics

Linear elastic fracture mechanics (LEFM) and nonlinear fracture mechanics (NLFM) models from (Červenka J. 1994) are chosen as particular representatives of the discrete crack model to be assessed in this work. The LEFM determines the crack propagation using the mode I and II stress intensity factors  $K_I, K_{II}$ , together with a prescribed value of fracture toughness  $K_{Ic}$  that can be estimated from the specific fracture energy:

$$K_{Ic} = \sqrt{E G_f} \quad (3)$$

where  $E$  is the elastic modulus.

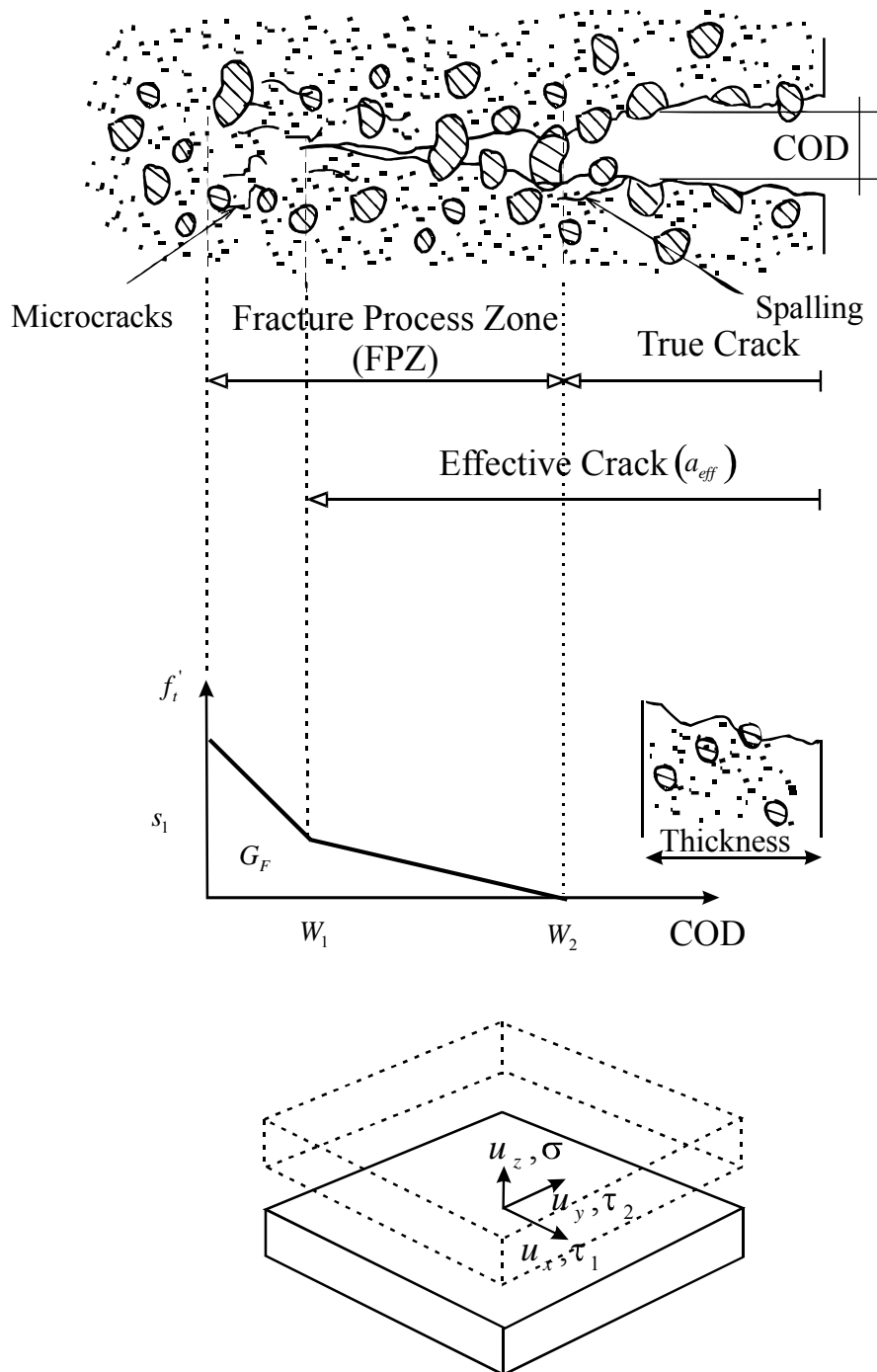


Fig. 3: Non-linear fracture mechanics – description of the theoretical assumptions and the interface model

Various criteria for the crack propagation have been proposed, and this work uses the maximum circumferential stress criterion of Erdogan and Shih (1963).

Whilst the LEFM approach assumes infinite stresses at the crack tip, the NLFM model considers a zone with cohesive stresses (i.e. fracture process zone) to exist ahead of the crack tip (see Fig. 3).

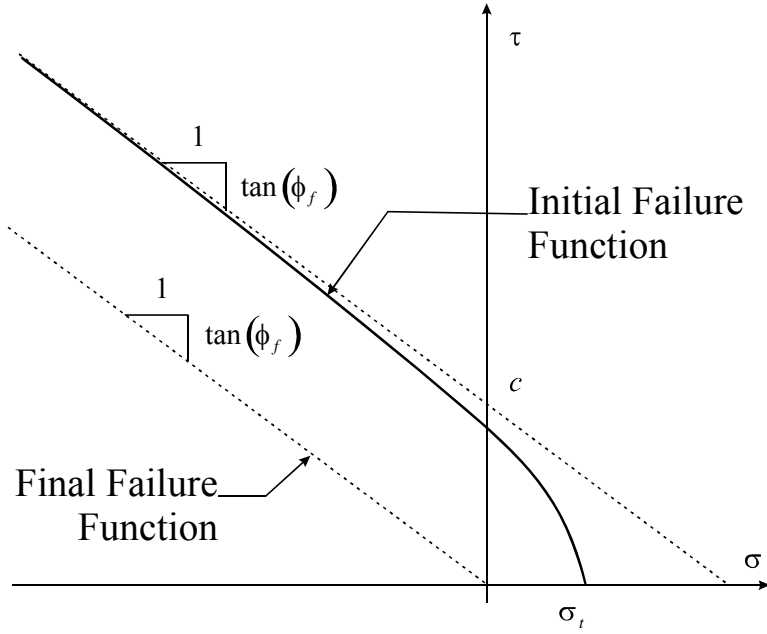


Fig. 4: The failure function for the interface crack model.

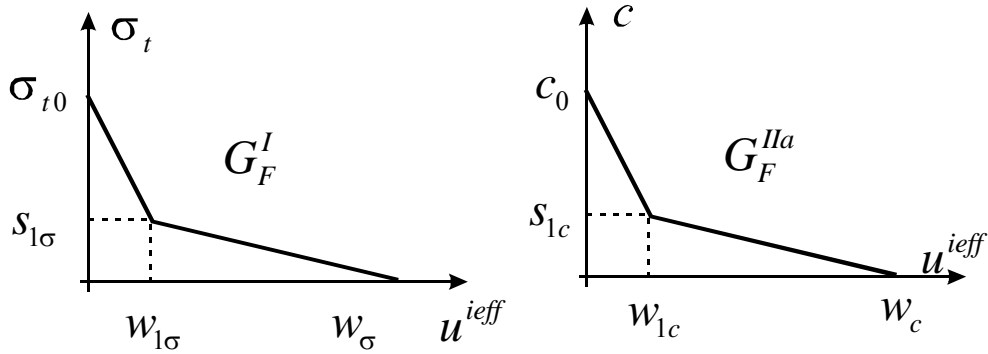


Fig. 5: The softening laws for the evolution of tensile strength and cohesion.

This cohesive zone can be modelled by interface elements with an appropriate constitutive law that relates the crack opening and sliding displacements to the normal and shear stresses. The program Merlin (1995) and the interface crack model proposed by Červenka J. (1994) is used in this comparative study. The basic equations and features of the interface crack model are summarized in the equations and figures below. For the formulation details the reader is referred to the original work.

The interface strength is governed by the failure envelope in Fig. 4. This envelope begins as a hyperbolic curve starting at the tensile strength  $\sigma_t$  and asymptotically approaching the line for dry friction.

$$F = (\tau_1^2 + \tau_2^2) - 2c \tan(\phi_f)(\sigma_t - \sigma) - \tan(\phi_f)^2 (\sigma^2 - \sigma_t^2) = 0 \quad (4)$$

where  $c$  and  $\phi_f$  are cohesion and material angle of friction, respectively.

During the cracking process this envelope reduces to the dry friction criterion using the evolution laws in Fig. 5.

$$c = c(u^{ieff}) \quad \sigma_t = \sigma_t(u^{ieff}) \quad (5)$$

The softening relations are based on the inelastic effective displacement:

$$\mathbf{u}^{ieff} = \|\mathbf{u}^i\| = \sqrt{(u_x^i)^2 + (u_y^i)^2 + (u_z^i)^2} \quad (6)$$

that is computed from the inelastic interface sliding and opening displacements, which are further decomposed in irreversible (i.e. plastic  $\mathbf{u}^p$ ) and reversible (i.e. fracturing  $\mathbf{u}^f$ ) components.

$$\mathbf{u} = \mathbf{u}^e + \mathbf{u}^i \quad \mathbf{u}^i = \mathbf{u}^p + \mathbf{u}^f \quad (7)$$

During the analysis of failure propagation, it is necessary to modify the finite element model, in order to introduce the new crack surfaces and interface elements. In program Merlin this is done semi-automatically. When crack propagation criteria are violated, the finite element analysis is terminated and the new crack propagation direction is calculated. Then the pre-processor is restarted, and new crack surfaces are added to the geometrical representation of the structure. After that, the finite element model is regenerated and the analysis is restarted from the beginning. The analysis progresses up to the point, when the crack propagation criteria are violated again, and this procedure of re-start and re-computation is repeated. The LEFM uses the maximum circumferential stress criterion of Erdogan and Shih (1963), whilst the NLFM adopts a method based on the value and direction of the maximal principal stresses at the crack tip.

## 4. Sample analyses

The second part of the paper presents three sample analyses of concrete and R/C structures. They were chosen to demonstrate advantages and drawbacks of smeared vs. discrete material models.

The 1<sup>st</sup> problem is the rather well known analysis of a four points shear beam used for testing shear strength of steel. It was originally proposed by Iosipescu (1967). The beam is solved analytically using both smeared and discrete cracks approaches, and the results are compared with experimental evidence.

The 2<sup>nd</sup> problem is an analysis of a reinforced shear wall with opening subject to lateral displacement at the top of the wall. Smeared crack approach was employed. The results are again compared with the experimental data.

The 3<sup>rd</sup> analysis is a pilot example of a combined approach in which smeared and discrete cracks models are combined in one finite element model.

### 4.1. Modified Iosipescu's shear beam

In this section a four point shear beam is analysed using both discrete and smeared crack models. This type of experiment was originally proposed by Iosipescu (1967) for testing of shear strength of steel, and later it was used by other researches (for instance Bažant and Pfeifer 1986) to study concrete materials. Originally, it was believed that this experiment can be used to determine the shear properties of concrete. However, later it was shown by Ingraffea and Pathanaki (1985) that the failure is governed by a pure mode I (tension) fracture.

Geometry of the specimen is shown in Fig. 6. The geometry and the material properties corresponds to the experiments that have been performed by Schlangen (1993). The tested beams have dimensions 440x100x100 mm. They were cast from concrete with modulus of elasticity  $E=35$  GPa, Poisson's ratio  $\nu=0.15$ , tensile strength  $f_t'=2.8$  MPa and the specific fracture energy  $G_F = 70$  N/m. The beam is first analysed by the discrete crack method using both linear and non-linear fracture mechanics. Then, the same beam is analysed by the smeared crack Sbeta material model. The results from the analyses are summarised in Fig. 7 to Fig. 10.

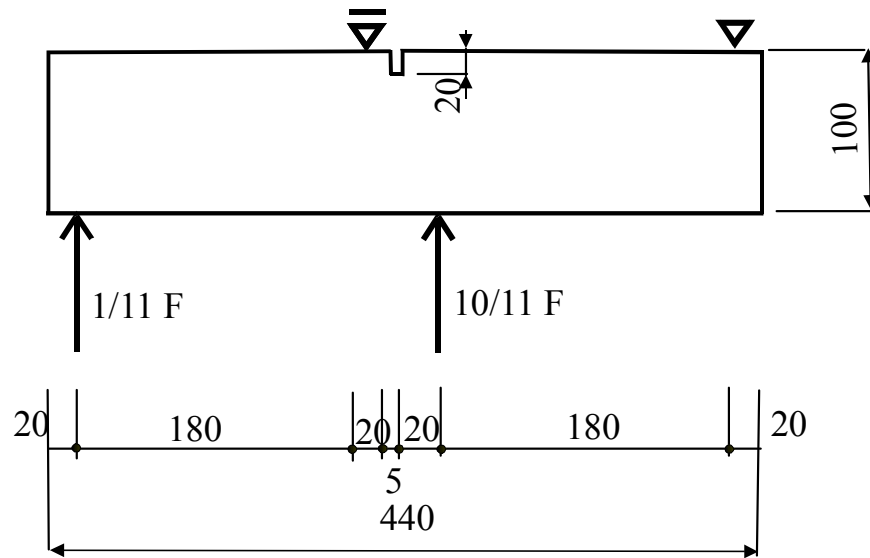
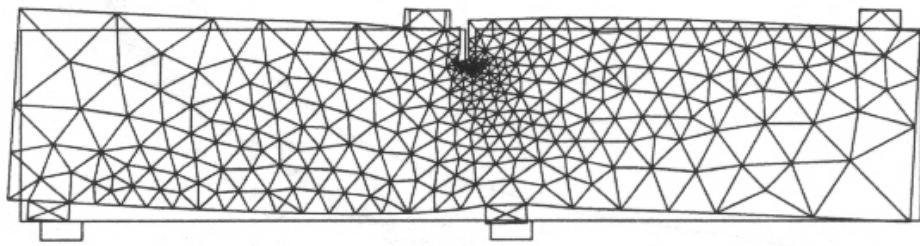


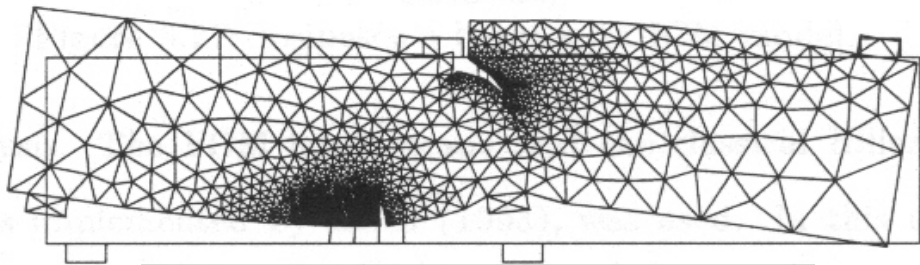
Fig. 6: Geometry of modified Iosipescu's beam

The results clearly indicate the main advantages and disadvantages of each approach. First of all, it is possible to observe that the linear elastic fracture mechanics is not applicable for this problem. This is mainly due to the small size of the structure. The criterion for crack propagation was based on the fracture toughness that was calculated from the specific fracture energy  $G_F$ . This assumption would be valid if the cohesive zone (i.e. fracture process zone) exists right from the beginning. For short crack length, however, the fracture process zone is not developed, and thus the energy necessary for the crack propagation is much lower. This explains why the peak load for LEFM is significantly overestimated. On the other hand, it is observed that the load-displacement curves (Fig. 10) for the discrete model using NLFM and the smeared crack model are almost identical, and match the experimental measurements. This observation is contradicted by the calculated crack path. In the discrete crack model, a curved crack starts at the notch and goes up to the right bottom support. Such failure mode is in a very good agreement with the experimental observations. This mode of cracking was not obtained in the smeared crack analysis (see Fig. 9), which shows a typical shear failure.

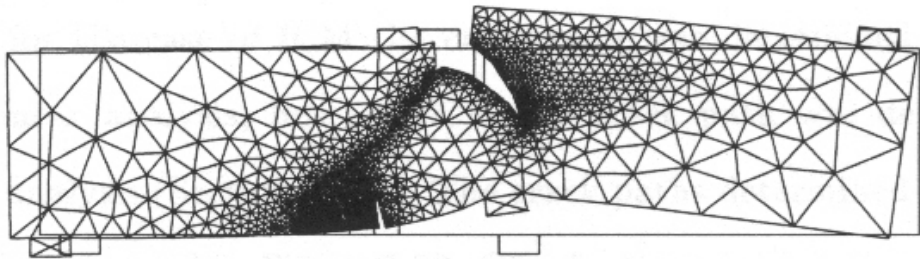
The computing time was higher for the discrete crack model analysis, since the analysis had to be restarted about 9 times before the final crack path was obtained. Due to the nature of the program Merlin (used for this analysis), user interventions were necessary to include the new crack extensions into the geometrical model. On the other hand the smeared crack analysis with the program Atena required about 15 minutes on a 800 MHz Pentium III computer and no user interaction was required at all.



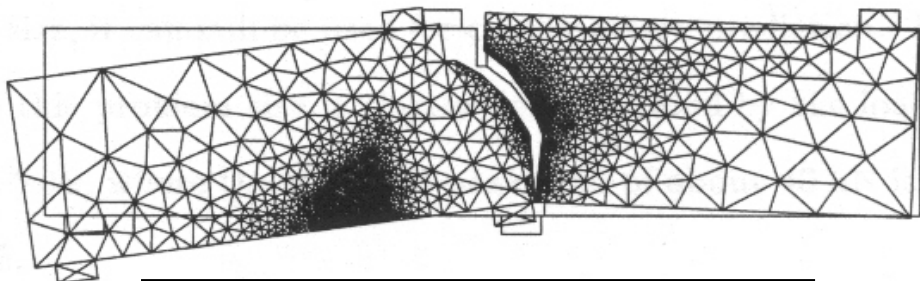
Deformed mesh at  $F = 25 \text{ kN}$



Deformed mesh at  $F = 40 \text{ kN}$



Deformed mesh at  $F = 25 \text{ kN}$  after the peak.



Deformed mesh at  $F = 0 \text{ kN}$  after the peak

Fig. 7: Crack propagation for multiple crack analysis and crack step size of 10 mm using discrete model and non-linear fracture mechanics.

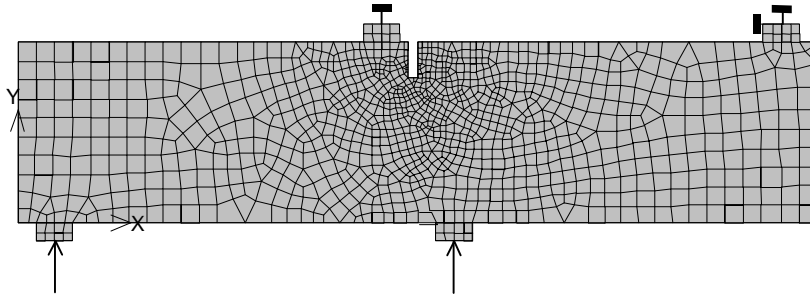


Fig. 8: The finite element mesh for Iosipescu's beam

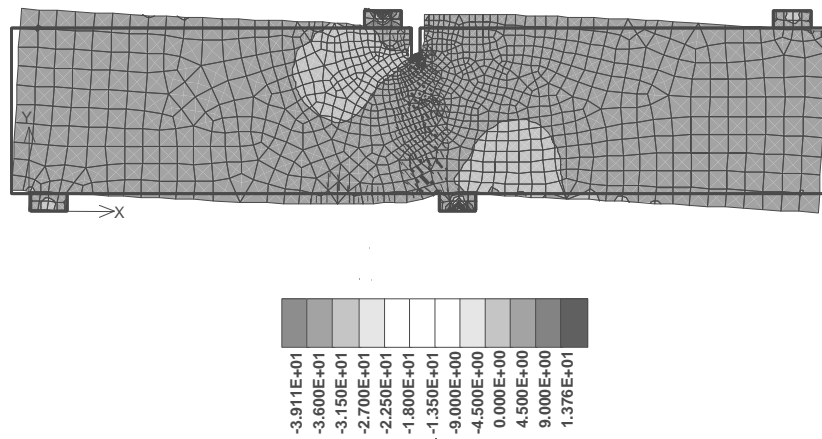


Fig. 9: Deformation, cracks and contours of maximum principal stress at peak load for Iosipescu's beam.

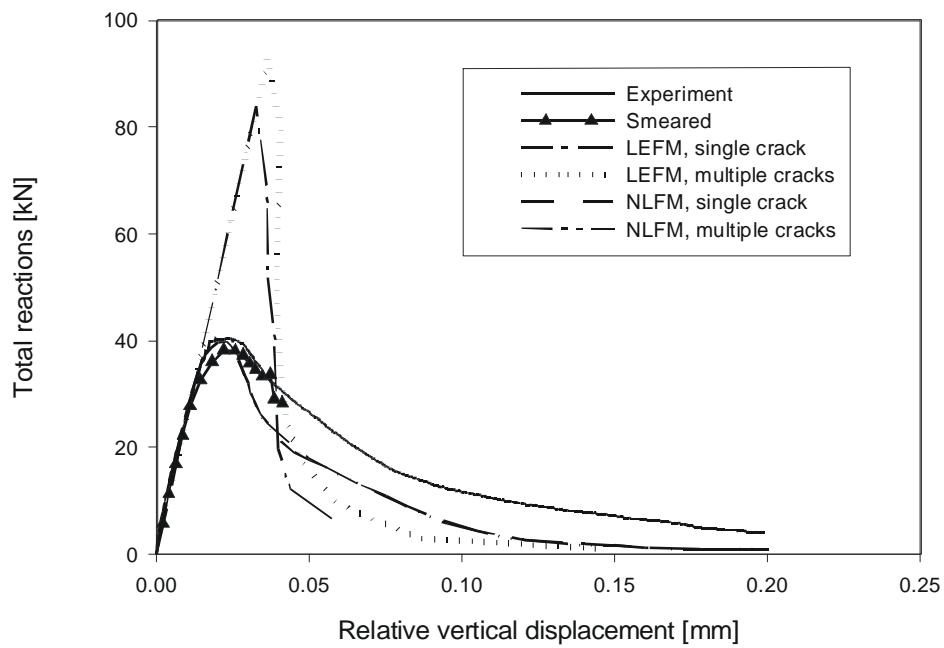


Fig. 10: Plot of the notch vertical opening vs. the total force.

## 4.2. Shear wall with openings

The second problem is a reinforced concrete wall with opening loaded in shear. This wall was tested in CEBTP institute in France (Fouré & Menzes, 1994). It was a benchmark problem for testing various simulation packages, and the Sbeta material model participated in this blind prediction study. The problem geometry is depicted in Fig. 11, and material properties for concrete and reinforcement are listed in Table 1 and Table 2. Two loading scenarios are investigated, namely the force A and the force B shearing the upper part of the wall to the right and left, respectively.

From the modelling point of view this example is rather complex. It contains smeared reinforcement in horizontal and vertical direction and several reinforcement bars that should be modelled by discrete truss elements. With significant effort, all these features can be modelled by standard finite element tools. In Atena, the smeared reinforcement is simulated by additional layers of elements with a special orthotropic constitutive formulation. The discrete reinforcement is included by truss elements with degrees of freedom tied to those of the intersected solid finite elements.

The results for the smeared crack calculation are summarised in Fig. 13 to Fig. 16

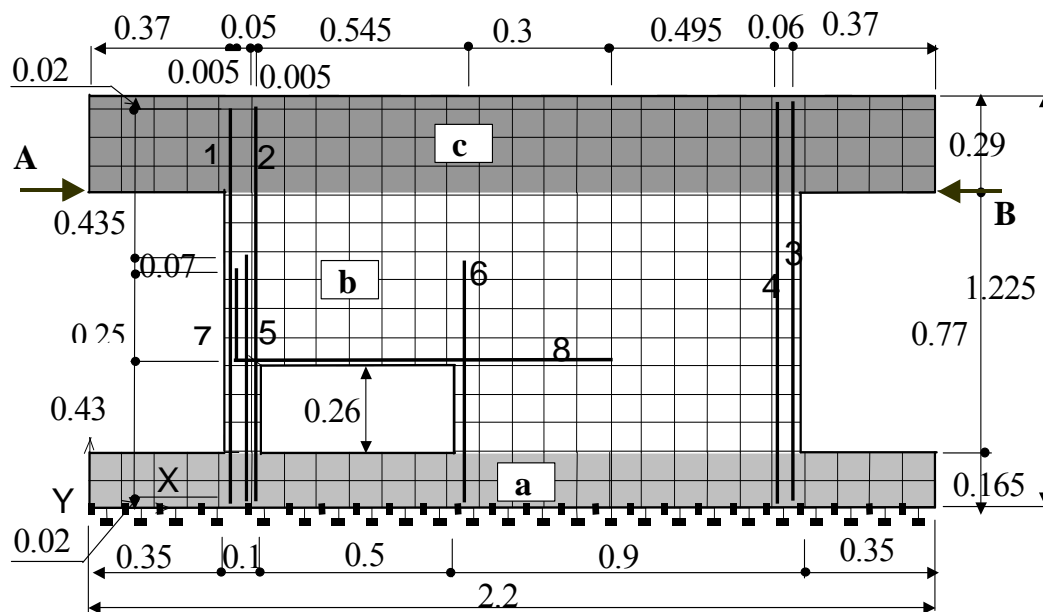


Fig. 11: Geometry and reinforcement of the wall

Table 1: Thickness and smeared reinforcement of the macro-elements

Region	Thickness	Material	Smeared reinforcement			
			Horizontal		Vertical	
			Ratio [%]	Material type	Ratio [%]	Material type
a	0.18m	1	0.16	3	0.28	3
b	0.09m	1	0.16	3	0.28	3
c	0.18m	2	0.16	3	0.28	3

Table 2: Material parameters for the shear wall analysis.

Material type 1	Sbeta material 1	Elastic material 2	Smeared Steel 3	Discrete Bar 2xd12 1-4	Discrete Bar d10 5-6	Discrete Bar d8 7-8
Elastic modulus E	31. GPa	44.5 GPa	200 GPa	200 GPa	200 GPa	200 GPa
Poisson's ratio $\nu$	0.200	0.200	N/A	N/A	N/A	N/A
Tensile strength $F_t$	2.6 MPa	N/A	N/A	N/A	N/A	N/A
Compressive strength $F_c$	-27.6 MPa	N/A	N/A	N/A	N/A	N/A
Type of tension softening	Hordijk	N/A	N/A	N/A	N/A	N/A
Specific fracture energy $G_f$	100 N/m	N/A	N/A	N/A	N/A	N/A
Hardening modulus	N/A	N/A	2.8 GPa	2.4 GPa	2.8 GPa	0.5 GPa
Yield strength	N/A	N/A	570 MPa	480 MPa	470 MPa	620 MPa

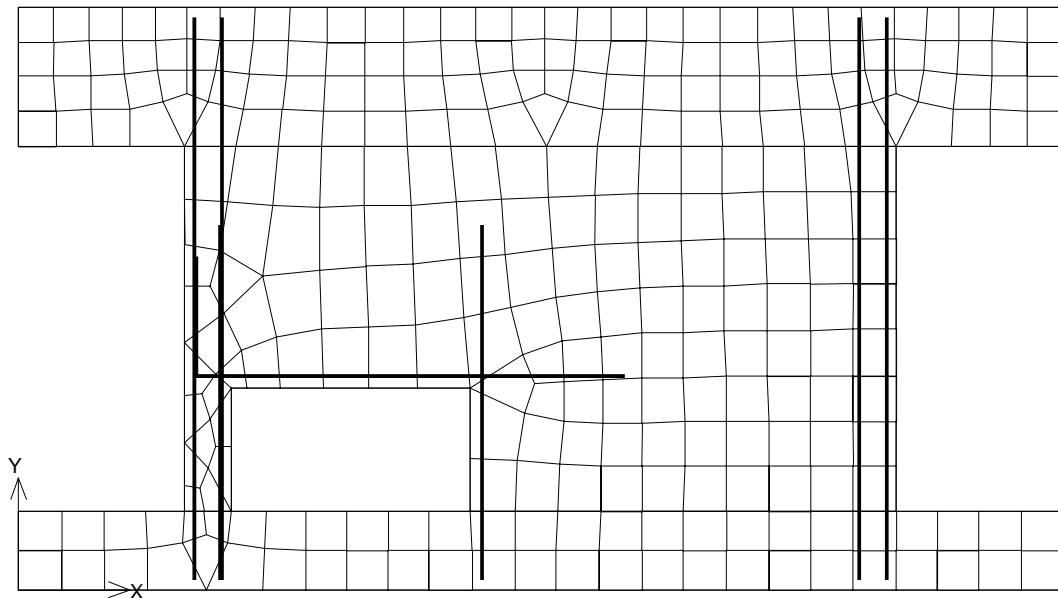


Fig. 12: Finite element mesh with discrete reinforcement bars that was used to analyse the wall.

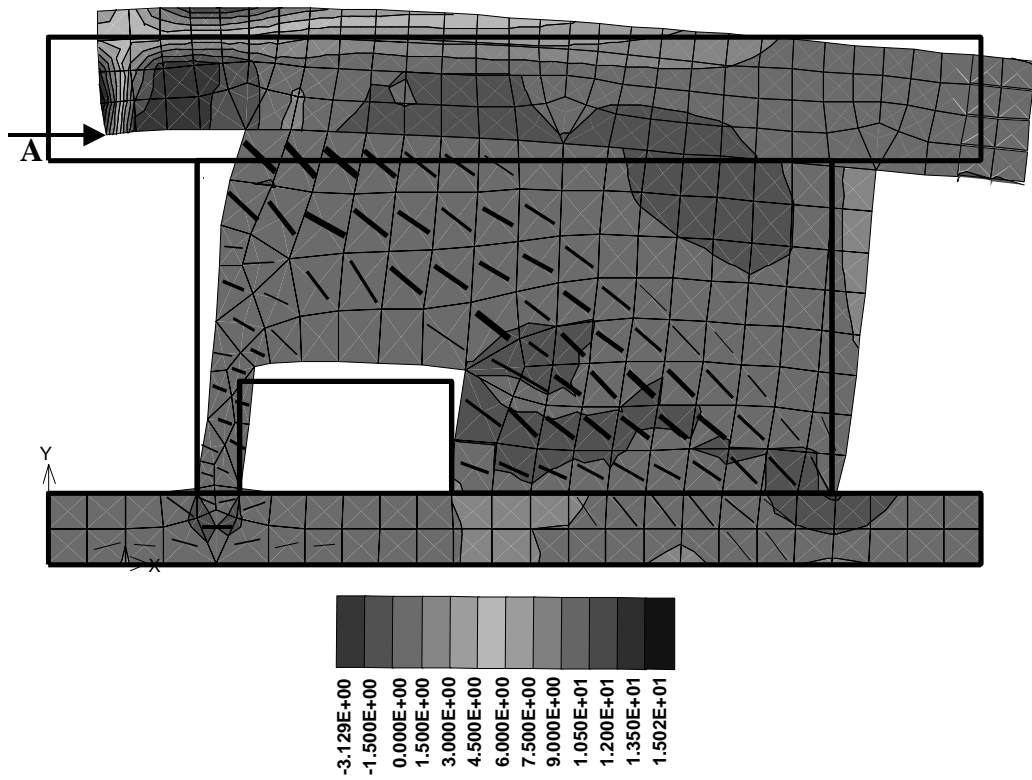


Fig. 13: Deformation, cracks and contours of maximum principal stress at peak loading of the wall loaded right by the force A.

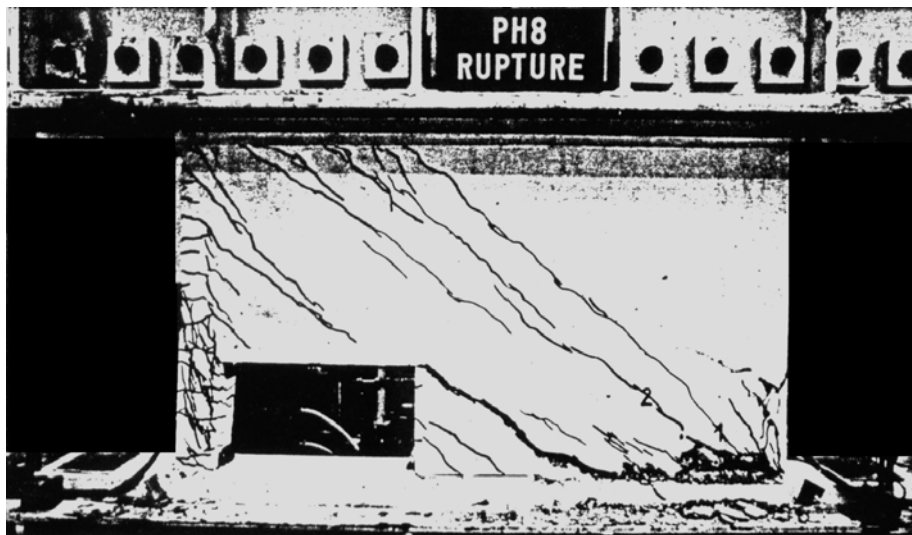


Fig. 14: Experimental results from CEBTP institute for the beam loaded by the force A.

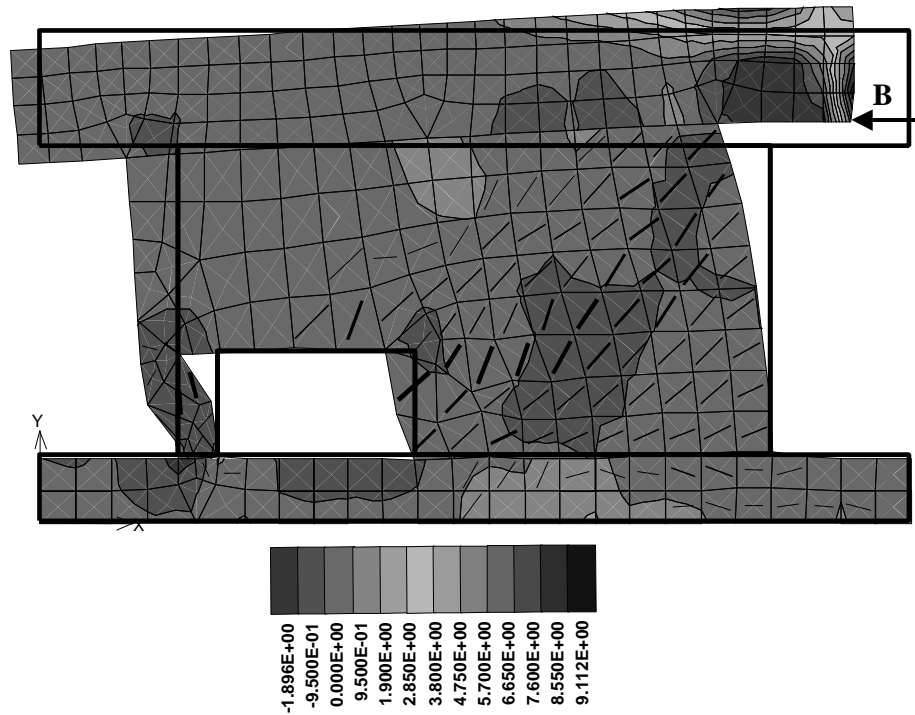


Fig. 15: Deformation, cracks and contours of maximum principal stress at peak loading of the wall loaded right by the force B

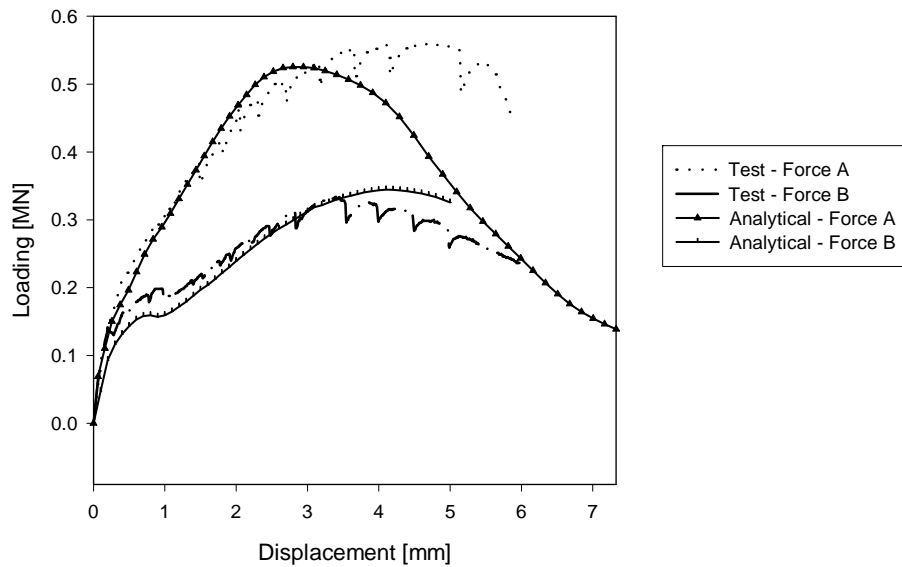


Fig. 16: Plot of loading force vs. displacement in direction of the applied force.

The numerical results show a very good agreement with the experiment in the pre-peak response and in the peak load. The post-peak results are affected by the three-dimensional effects of the tie-rods, which are very difficult to capture in a two-dimensional analysis.

The confinement due to the tie-rods has a strong affect on the compressive softening in the zones, where concrete crushing takes part.

This example presents a rather complex problem, where the structural failure involves concrete cracking, crushing and reinforcement yielding. This analysis is a perfect example of a problem that would be almost impossible to simulate by discrete crack model. The authors do not have a software tool suitable for modelling the different reinforcement types in this problem in the combination with the discrete crack model. Before the failure localisation, the cracking is very distributed, as it can be seen from Fig. 14., thus many discrete cracks would have to be inserted and propagated in the model to simulate the reality. In addition, the concrete crushing, which is a dominant failure mode in this example, cannot be modelled by interface elements.

### 4.3. The combined smeared-discrete crack analysis

The first two sample problems document the advantages and disadvantages of the both approaches. The third example presents a pilot analysis attempting to adopt a combined approach, where the smeared crack model is used to model the distributed cracking and concrete crushing, while the discrete cracks are inserted into the areas, where a major crack can be expected on the basis of an engineering judgement.

Geometry of the problem and material properties correspond to the column tested by Xiao et. al. (1993). The geometry and reinforcement layout is shown in Fig. 17 and the experimental setup and crack pattern after the cycling loading is depicted in Fig. 19. The problem was analysed by the program Merlin (see Merlin 1995) using the three-dimensional extension of the smeared crack model that is in detail described in Červenka J. et. al. (1994) Fig. 18 shows the crack pattern, deformed shape and contour areas of the equivalent plastic strains at the last step in the analysis after one cycle.

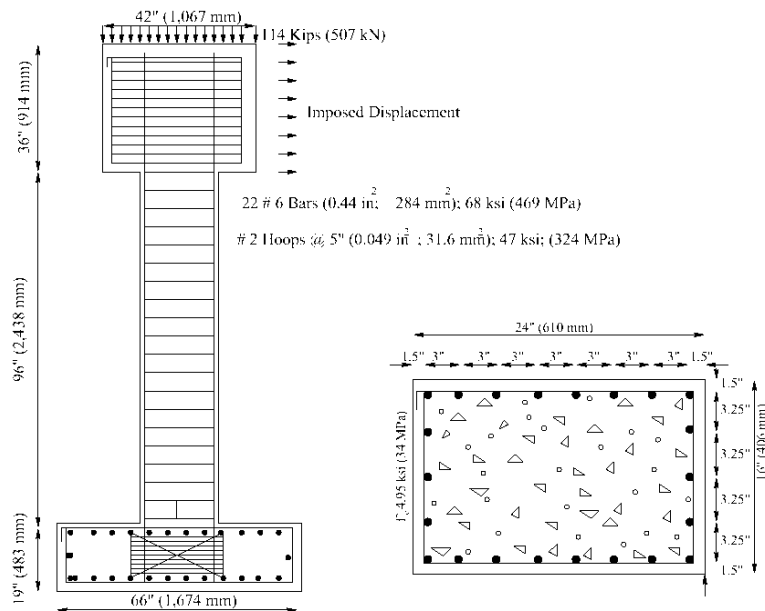


Fig. 17: Geometry of the reinforced concrete column tested by California Department of Transportation.

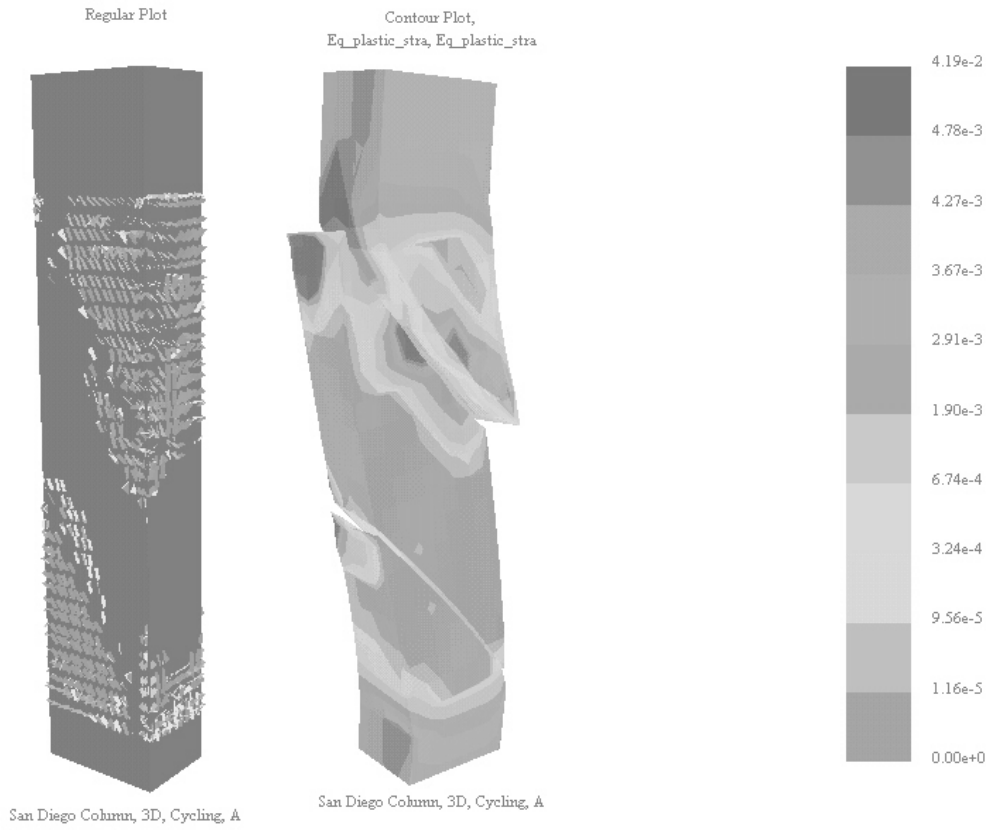


Fig. 18: The results from the combined smeared-discrete crack analysis.

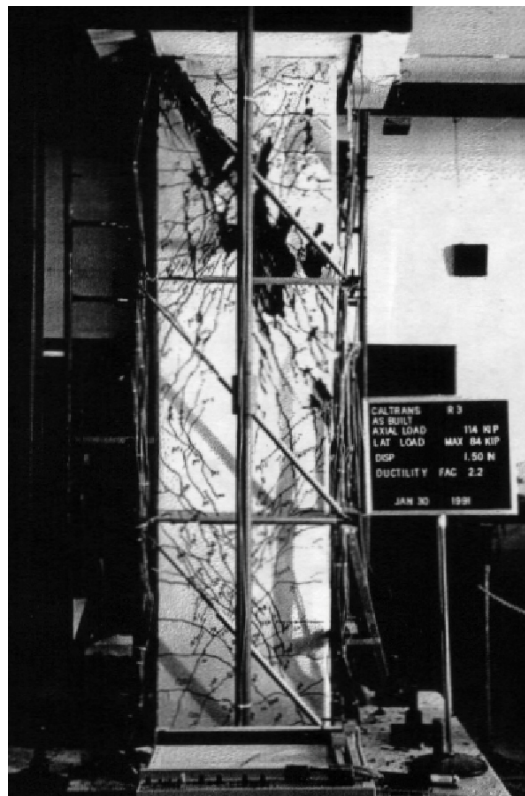


Fig. 19 The experimental results by Xiao et. al. (1993).

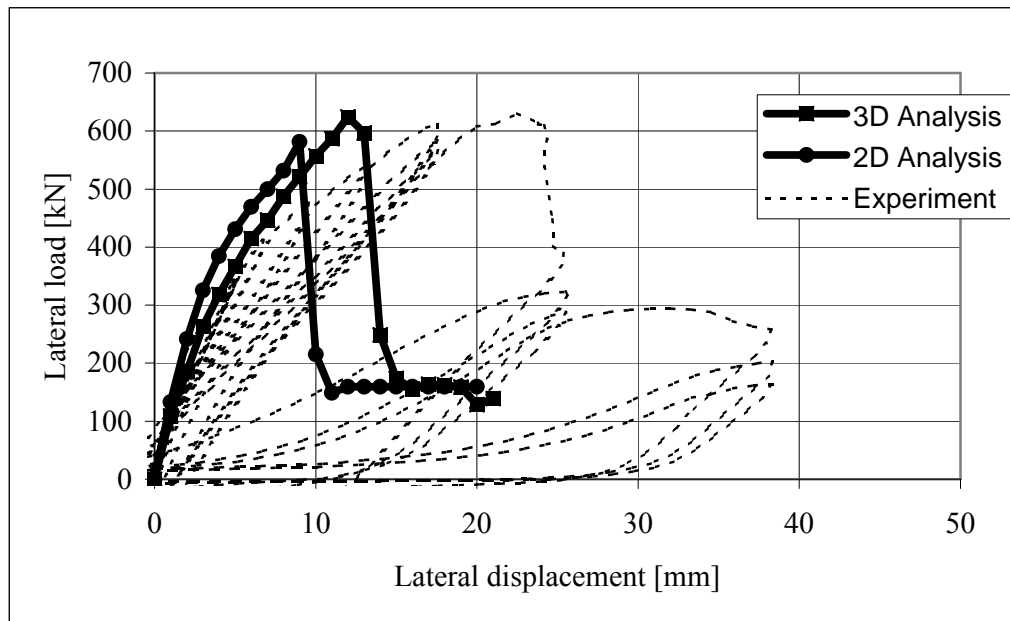


Fig. 20 The comparison of load-displacement curves for two and three-dimensional analyses.

The results show a good agreement in the peak load and final failure mode. There are differences in the post-peak response and peak displacement that can be attributed to the usage of a relatively coarse mesh in this pilot analysis. The combined approach, nevertheless, shows a lot of potential in combining the positive features of the two methods and eliminating their disadvantages. The first experience with this kind of analysis shows that the model preparation is rather demanding, but does not require any special features in the finite element solver or data-preparation program. It should be possible to prepare such a model with standard tools available on the market, but requires some tedious work to insert the reinforcement rods. This can be simplified by various special features for the automation of the reinforcement generation, which are usually not present in general purpose FE pre-processors, but are available in special packages like the employed programs Atena and Merlin. The current results show that the combined approach enables to capture the failure mode and crack pattern very precisely, but the accuracy of the overall response seems to be dominated by the performance of the smeared crack material model.

## 5. Conclusion

The basic features of the smeared and discrete crack models have been presented and discussed using two typical representatives for each family of the models. After that, two example analyses were presented. The first example was a plain concrete beam subject to shear and it demonstrated the accuracy of the discrete crack model in predicting the correct failure crack path. Nevertheless, both models calculated correct pre-peak and peak behaviour of the beam. The second example was a reinforced concrete wall subject to shear load at its top. This relatively complex problem, which involved the concrete cracking, crushing and reinforcement yielding, documented the superiority of the smeared crack model. This example, which represents a typical problem from the engineering practice, can be solved by the discrete crack method only with an extensive effort. The third example was a pilot analysis combining the above two approaches. It can be recommended for cases, when pure smeared crack models fail to localise properly and the modelling of the failure localisation is an important feature for the overall structural response.

Based on the described sample problems and on the authors' experience the following recommendations are summarized in Table 3. The problems are categorized according to two criteria: first one is the presence of reinforcement and the second is the possibility to estimate the location and direction of the failure localization.

Table 3: The recommendations for smeared and discrete crack models usage in the analysis of concrete structures.

	Plain concrete	Reinforced concrete
Known crack path	discrete or smeared	smeared
Unknown crack path	discrete or smeared	combined or smeared

It is clear from the above table that the smeared crack model is currently the most general tool for the practical engineering problems while the discrete model is the preferred choice for the analysis of structures made of plain or slightly reinforced concrete.

In the case of the discrete crack model, the engineer should also realise the two available choices: linear or non-linear fracture mechanics. Massive concrete structures like for instance dams can be very effectively analysed using linear fracture, while non-linear mechanics appears to be a more general tool provided a sufficiently fine mesh is used for large structures.

## 6. Acknowledgements

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