Fracture-Plastic Material Model for Concrete

Theoretical background

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Fracture Plastic Model

Fundamentals

- Strain decomposition:

Plastic strain

$$\varepsilon_{ij} = \varepsilon_{ij}^{e} + \varepsilon_{ij}^{p} + \varepsilon_{ij}^{f} \qquad (1)$$

Fracturing strain

- New stress state calculation

$$\sigma_{ij}^{n} = \sigma_{ij}^{n-1} + E_{ijkl}(\Delta\varepsilon_{kl} - \Delta\varepsilon_{kl}^{p} - \Delta\varepsilon_{kl}^{f}) \qquad (2)$$

Rankine Softening Model in Tension

Fracture Plastic Model

- Rankine criterion

$$F_i^{f} = \sigma_{ii}^{n'} - f'_{t\,i} = \sigma_{ii}^{t'} - E_{iikl}\,\Delta\varepsilon_{kl}^{f'} - f'_{t\,i} = 0 \qquad (3)$$

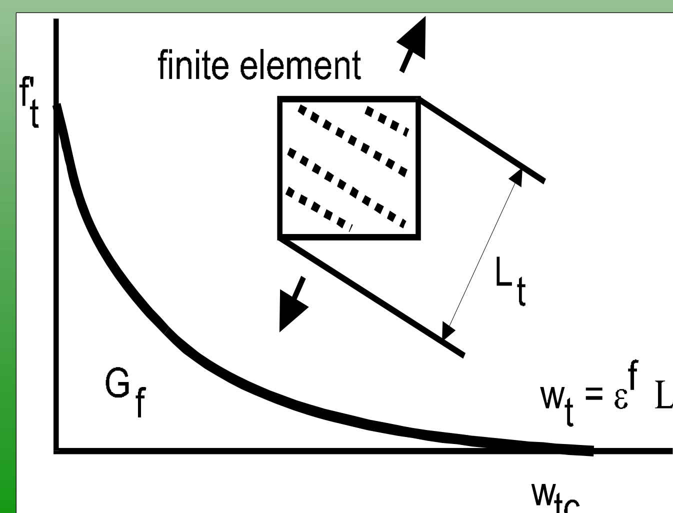
Tensile strength ($f'_{t\,i}$); Trial stress ($\sigma_{ii}^{t'}$); Fracturing strain ($\Delta\varepsilon_{kl}^{f'}$)

- Fracturing strain increment in direction *k*

Two equations solved iteratively

$$\Delta\varepsilon_{kk}^{f'} = \frac{\sigma_{kk}^{t'} - f'_{tk}}{E_{kkkk}} = \frac{\sigma_{kk}^{t'} - f'_{tk}(w)}{E_{kkkk}}$$

$$w = L_t\left(\hat{\varepsilon}_{kk}^{f'} + \Delta\varepsilon_{kk}^{f'}\right) \qquad (4)$$



Fracture Plastic Model

Rankine Fracturing Model Matrices

- Secant constitutive relationship in matrix form (Rots 1989)

$$\mathbf{s}' = (\mathbf{E} - \mathbf{E}(\mathbf{E}^{cr} + \mathbf{E})^{-1}\,\mathbf{E}\,)\,\mathbf{e}'$$

Stress vector

Strain vector

- Current fracturing strains

$$\mathbf{e}^{f\,'} = (\mathbf{E}^{cr} + \mathbf{E})^{-1}\,\mathbf{E}\,\mathbf{e}'$$

Fracturing strain

Fracturing matrix $\mathbf{E}^{cr}$ defined as $\mathbf{s} = \mathbf{E}^{cr}\,\mathbf{e}^{f\,'}$

Fracture Plastic Model

Plasticity Model for Crushing

- Predictor-corrector formula

Elastic predictor

$$\sigma_{ij}^{n} = \sigma_{ij}^{n-1} + E_{ijkl}(\Delta\varepsilon_{kl} - \Delta\varepsilon_{kl}^{p}) = \sigma_{ij}^{t} - E_{ijkl}\Delta\varepsilon_{kl}^{p} = \sigma_{ij}^{t} - \sigma_{ij}^{p} \qquad (5)$$

Plastic corrector

- Stress σ_{ij}^{p} from yield function

$$F^{p}(\sigma_{ij}^{t} - \sigma_{ij}^{p}) = F^{p}(\sigma_{ij}^{t} - \Delta\lambda l_{ij}) = 0 \qquad (6)$$

- Return direction

$$l_{ij} = E_{ijkl}\frac{\partial G^{p}(\sigma_{kl}^{t})}{\partial \sigma_{kl}} \quad \text{then} \quad \Delta\varepsilon_{ij}^{p} = \Delta\lambda \frac{\partial G^{p}(\sigma_{ij}^{t})}{\partial \sigma_{ij}} \qquad (7)$$

Fracture Plastic Model

Yield/Failure Functions in Plasticity Model

- Drucker-Prager

$$F^{p}_{DP}(\sigma_{ij}) = \alpha\, I_1 + \sqrt{J_2} - k = 0 \qquad (8)$$

- Three-parameter Menetrey-Willam (ACI 1995) (f'_c, f'_t, e)

$$F^{p}_{3P} = \left[\sqrt{1.5}\,\frac{\rho}{f'_c}\right]^2 + m\left[\frac{\rho}{\sqrt{6}\,f'_c}\, r(\theta,e) + \frac{\xi}{\sqrt{3}\,f'_c}\right] - c = 0 \qquad (9)$$

$$m = \sqrt{3}\,\frac{f_c'^2 - f_t'^2}{f'_c\, f'_t}\,\frac{e}{e+1}$$

$$r(\theta,e) = \frac{4(1-e^2)\cos^2\theta + (2e-1)^2}{2(1-e^2)\cos\theta + (2e-1)\left[4(1-e^2)\cos^2\theta + 5e^2 - 4e\right]^{\frac{1}{2}}}$$

Return Direction, Plastic Potential

- Plastic corrector direction

$$G^p(\sigma_{ij}) = \beta \frac{1}{\sqrt{3}} I_1 + \sqrt{2 J_2}$$

β Parameter defines the amount of dilatancy

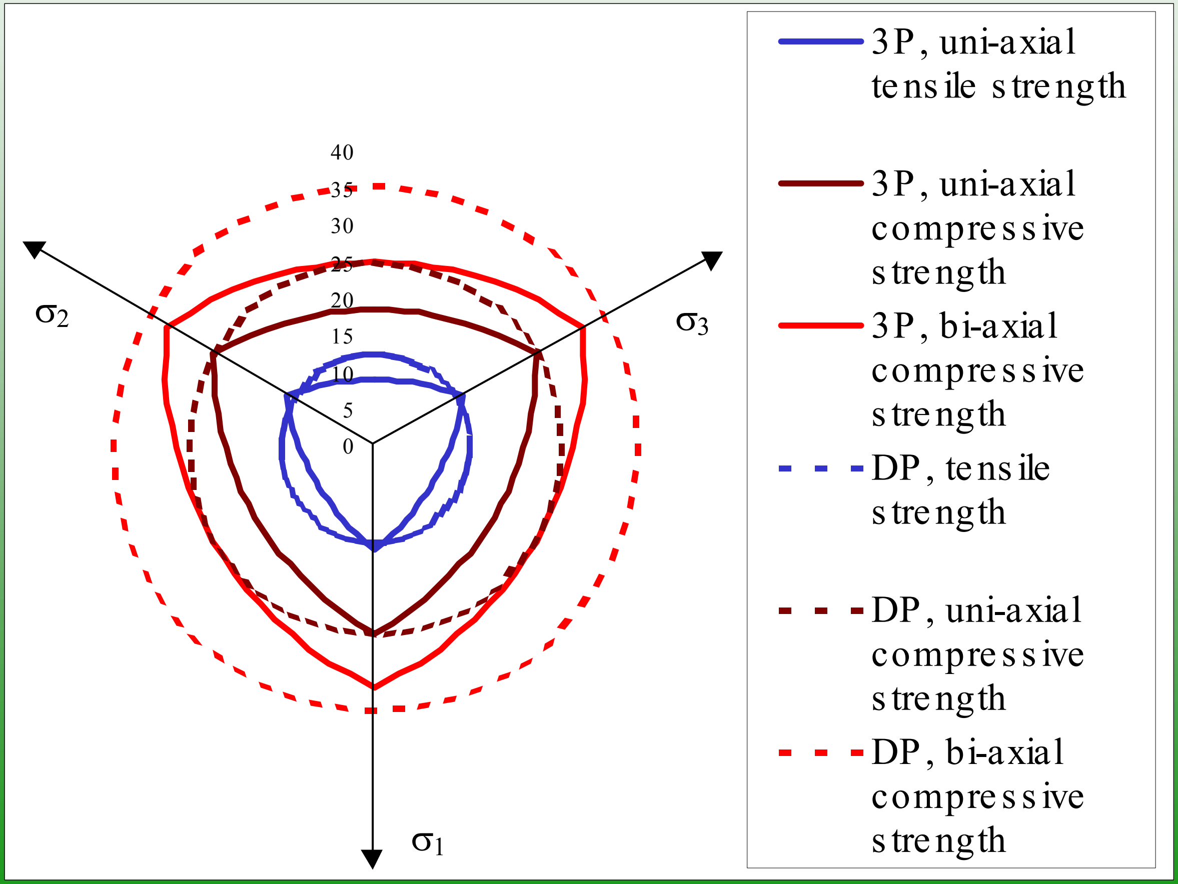
β < 0 compaction
β = 0 volume is preserved
β > 0 volume is increasing

Equal to Haigh-Vestergaard coordinate ξ

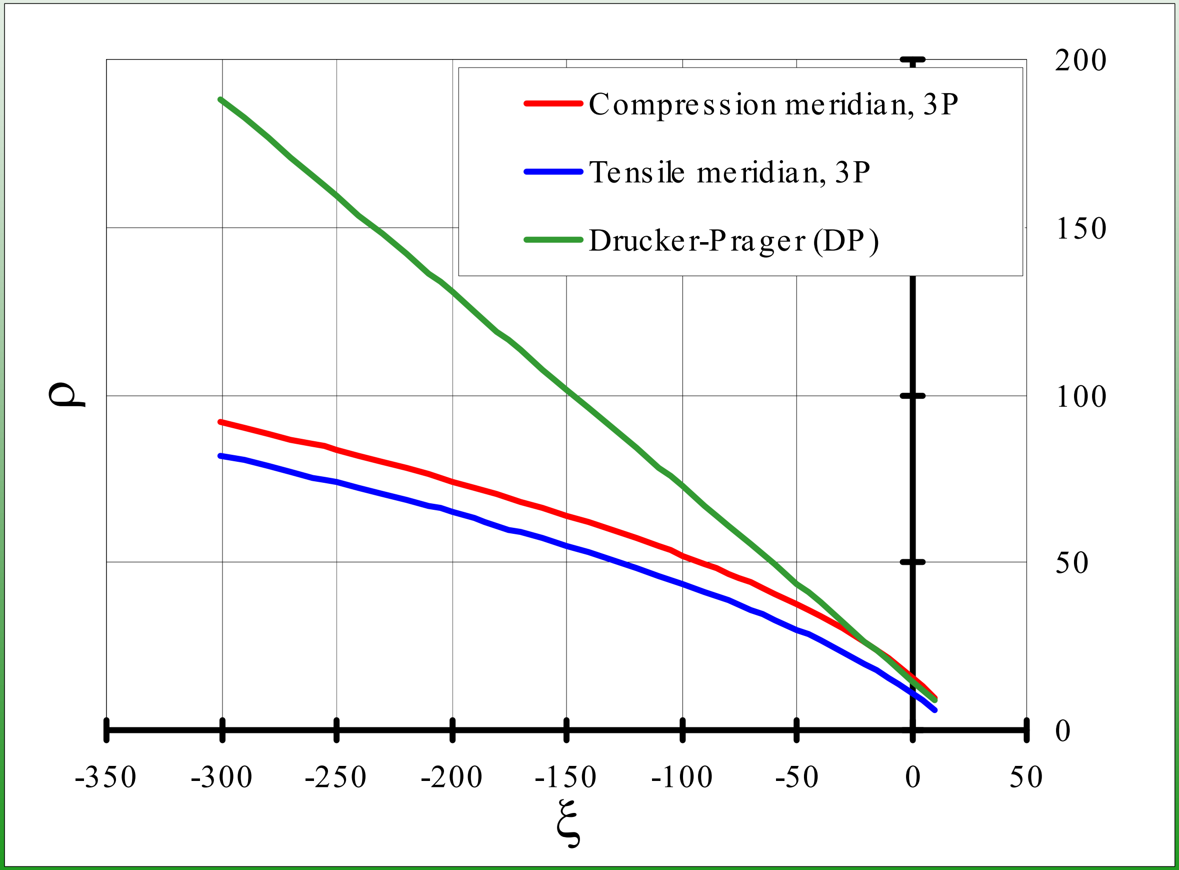
Equal to Haig-Vestergaard coordinate ρ

Fracture
Plastic
Model

Plastic Failure Functions Deviatoric Plane



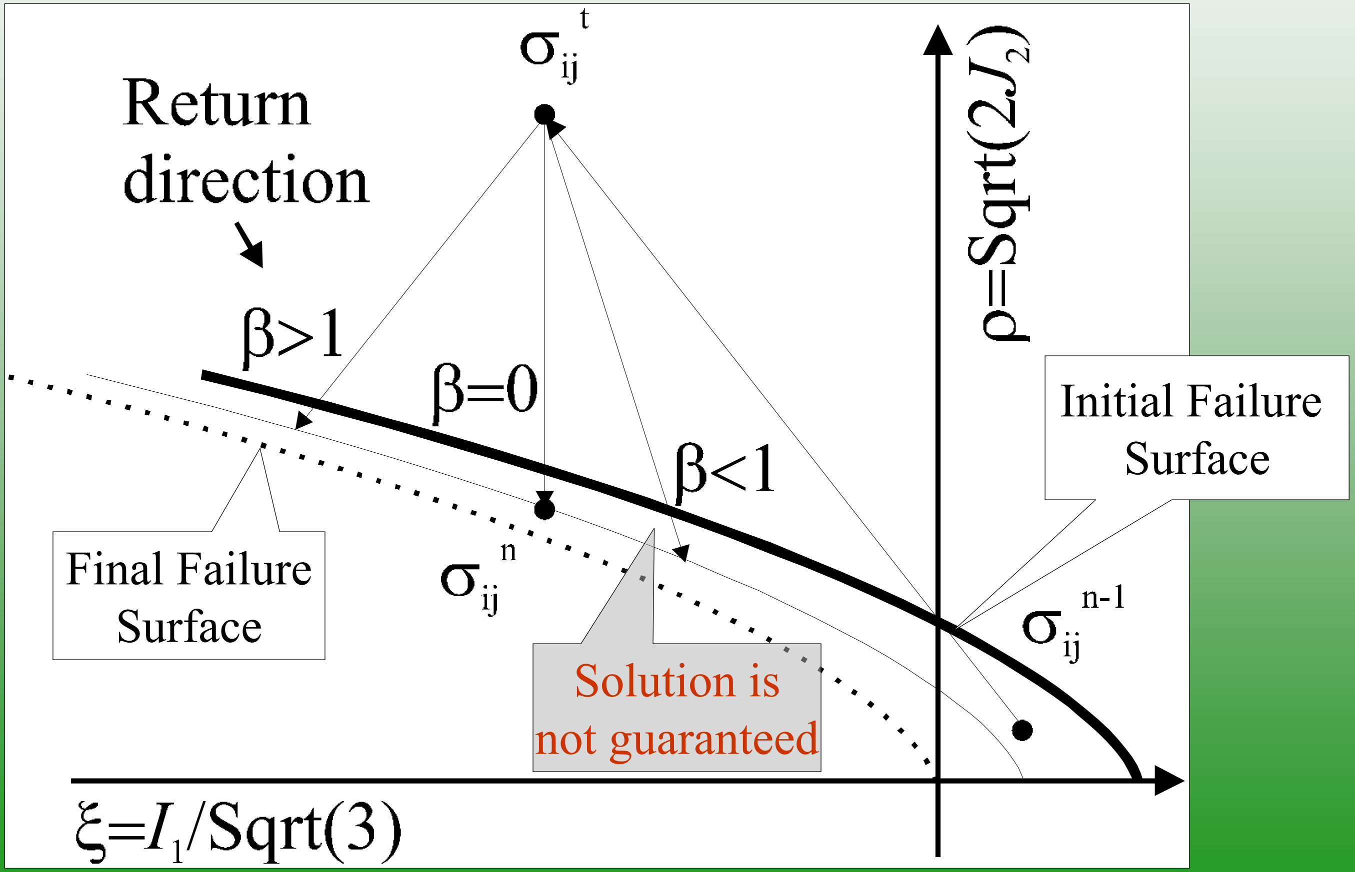
Plastic Failure Functions in ξ–ρ Plane



Fracture Plastic Model

Plasticity Predictor-Corrector Scheme

Fracture Plastic Model



Fracture Plastic Model

Combination of Plasticity and Fracture Model

- Simultaneous solution of the following two inequalities

Plastic model

$$F^{p}(\sigma_{ij}^{n-1} + E_{ijkl}(\Delta\varepsilon_{kl} - \Delta\varepsilon_{kl}^{f} - \Delta\varepsilon_{kl}^{p})) \leq 0$$ solve for $\Delta\varepsilon_{kl}^{p}$

Fracturing model

$$F^{f}(\sigma_{ij}^{n-1} + E_{ijkl}(\Delta\varepsilon_{kl} - \Delta\varepsilon_{kl}^{p} - \Delta\varepsilon_{kl}^{f})) \leq 0$$ solve for $\Delta\varepsilon_{kl}^{f}$

Iterative Scheme for Combined Model

Step 1:

$$F^{p}(\sigma_{ij}^{n-1} + E_{ijkl}(\Delta\varepsilon_{kl} - \Delta\varepsilon_{kl}^{f\,(i-1)} + b\Delta\varepsilon_{kl}^{cor\,(i-1)} - \Delta\varepsilon_{kl}^{p\,(i)})) \le 0 \quad \text{solve for} \quad \Delta\varepsilon_{kl}^{p\,(i)}$$

Step 2: Reversed with step 1 if stress from step 1 violates fracturing criterion.

$$F^{f}(\sigma_{ij}^{n-1} + E_{ijkl}(\Delta\varepsilon_{kl} - \Delta\varepsilon_{kl}^{p\,(i)} - \Delta\varepsilon_{kl}^{f\,(i)})) \le 0 \quad \text{solve for} \quad \Delta\varepsilon_{kl}^{f\,(i)}$$

Step 3:

$$\Delta\varepsilon_{ij}^{cor\,(i)} = \Delta\varepsilon_{ij}^{f\,(i)} - \Delta\varepsilon_{ij}^{f\,(i-1)}$$

b - based on the analysis of α^{p} and α^{f}

$$\alpha^{p} = \frac{\left\| \Delta\varepsilon_{ij}^{p\,(i)} - \Delta\varepsilon_{ij}^{p\,(i-1)} \right\|}{\left\| \Delta\varepsilon_{ij}^{cor} \right\|}$$

$$\alpha^{f} = \frac{\left\| \Delta\varepsilon_{ij}^{f\,(i)} - \Delta\varepsilon_{ij}^{f\,(i-1)} \right\|}{\left\| \Delta\varepsilon_{ij}^{p\,(i)} - \Delta\varepsilon_{ij}^{p\,(i-1)} \right\|}$$

Iterative correction:

$$\left\| \Delta\varepsilon_{ij}^{cor\,(i)} \right\| = (1-b)\,\alpha^{f}\,\alpha^{p} \left\| \Delta\varepsilon_{ij}^{cor\,(i-1)} \right\|$$

Must be < 1 to guarantee convergence

Fracture Plastic Model

Iteration Algorithm Analysis

Iterative correction: $\left\| \Delta\varepsilon_{ij}^{cor\,(i)} \right\| = (1-b)\ \alpha^{f}\ \alpha^{p} \left\| \Delta\varepsilon_{ij}^{cor\,(i-1)} \right\|$

Convergence condition: $\left| (1-b)\alpha^{f}\ \alpha^{p} \right| < 1$

b - depends on stress state, dilation and material brittleness

Analysis of iteration parameters α^{p} and α^{f}:

$$\alpha^{p} = \frac{\left\| \Delta\varepsilon_{ij}^{p\,(i)} - \Delta\varepsilon_{ij}^{p\,(i-1)} \right\|}{\left\| \Delta\varepsilon_{ij}^{cor} \right\|}$$

- $\alpha^{p} = 0$ if plasticity not activated
- $\alpha^{p} < 1$ hardening material without dilation
- $\alpha^{p} \geq 1$ softening or dilating material

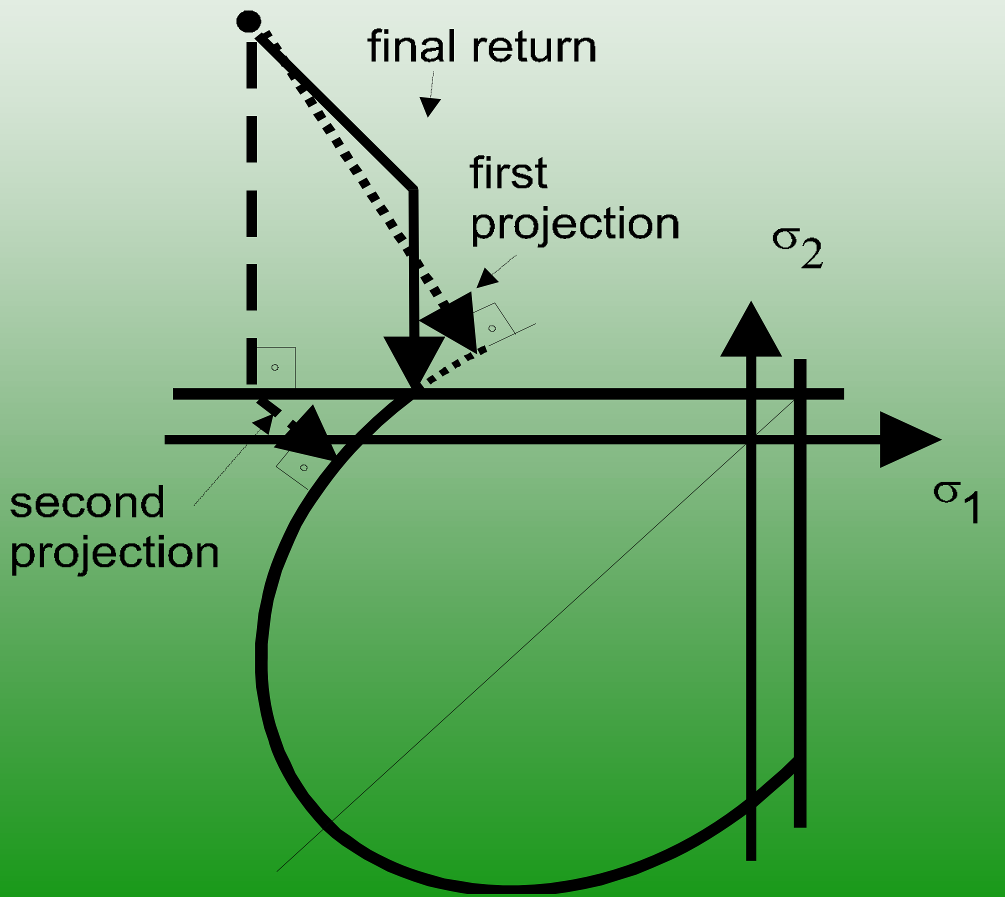
$$\alpha^{f} = \frac{\left\| \Delta\varepsilon_{ij}^{f\,(i)} - \Delta\varepsilon_{ij}^{f\,(i-1)} \right\|}{\left\| \Delta\varepsilon_{ij}^{p\,(i)} - \Delta\varepsilon_{ij}^{p\,(i-1)} \right\|}$$

- $\alpha^{f} = 0$ if fracturing not activated
- $\alpha^{f} < 1$ in case of crack closing
- $\alpha^{f} \geq 1$ in case of softening

Fracture Plastic Model

Schematic description of the iterative process

(for simplicity only in 2D)

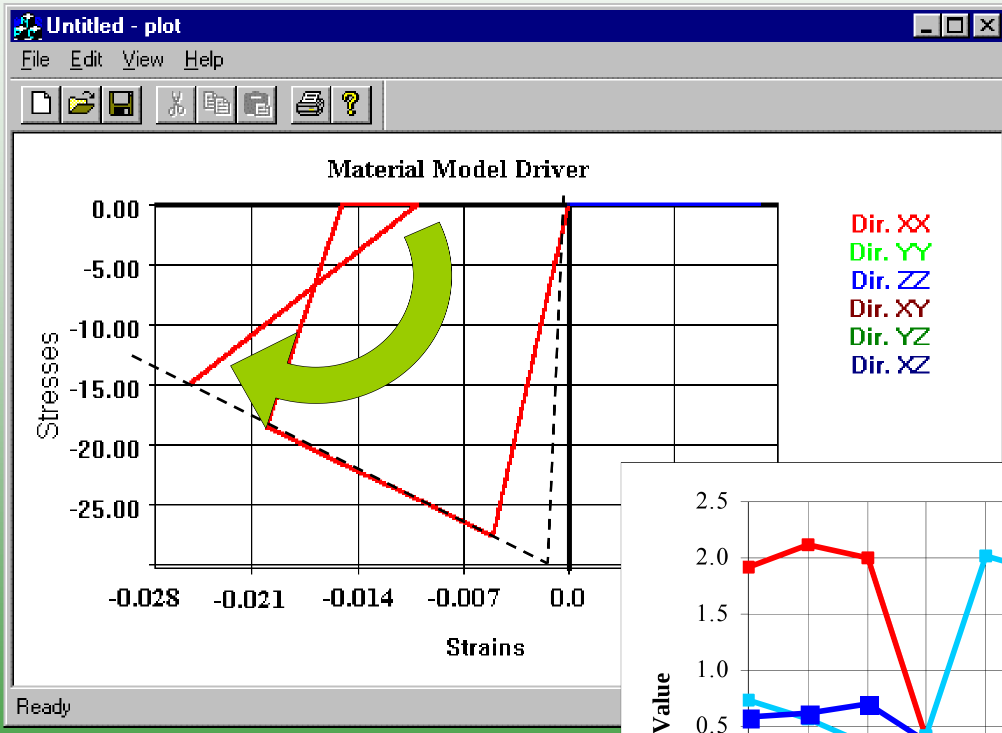


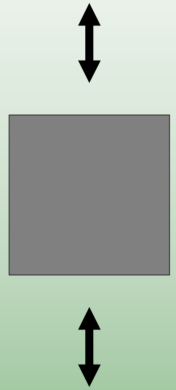
Fracture-Plastic Material Model Conclusions

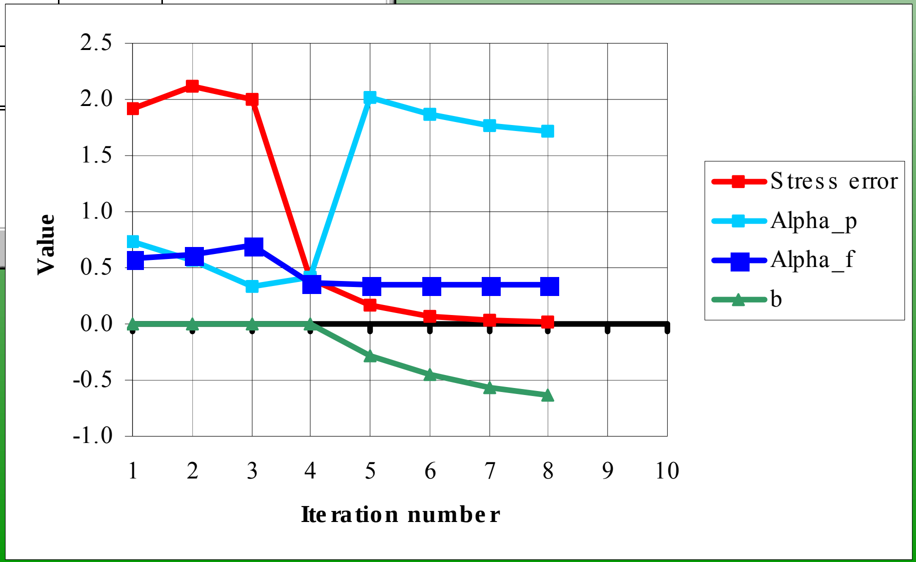
- Total separation of fracture and plastic model advantageous for object oriented programming.
- Predictor-corrector scheme guarantees robust plastic model if $\beta \geq 0$.
- Robust iterative algorithm was developed for combining fracture and plastic models.

Fracture Plastic Model

Uni-axial Compression with Un- and Re-loading

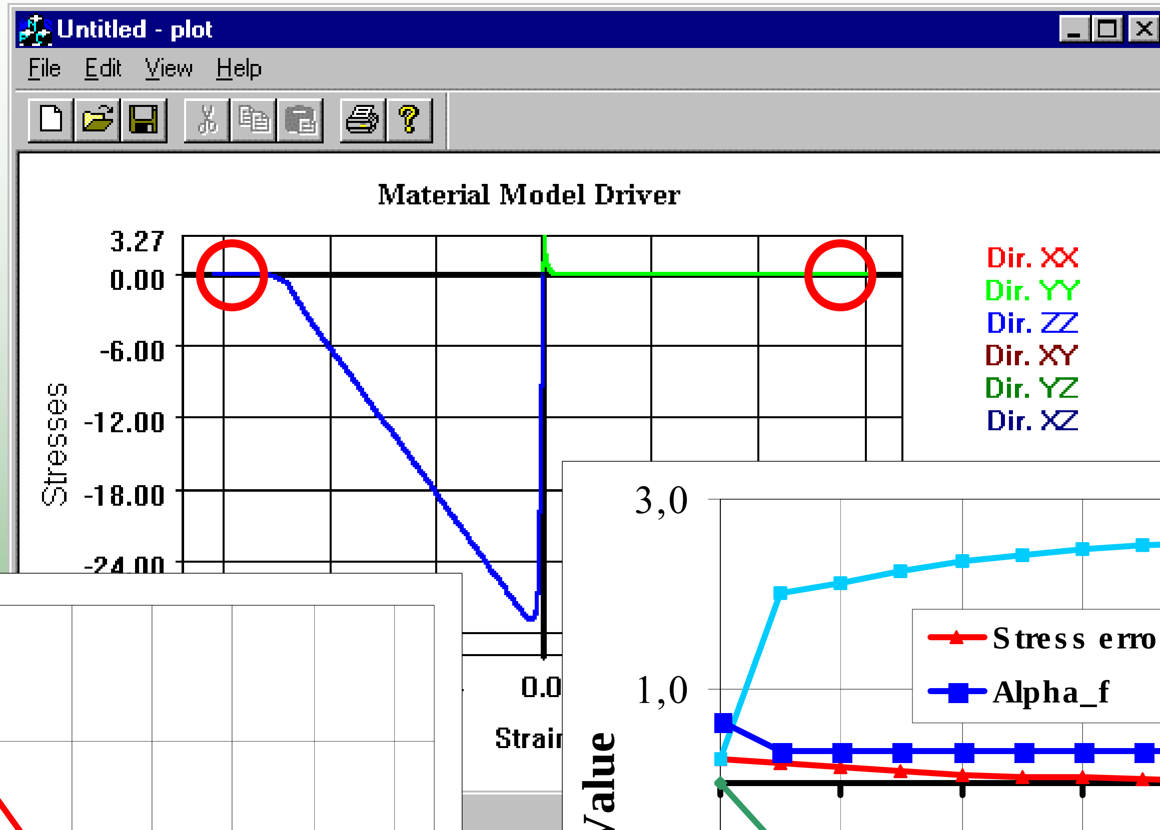


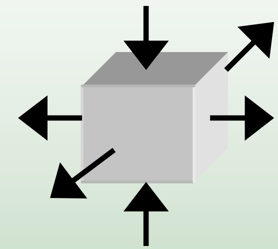


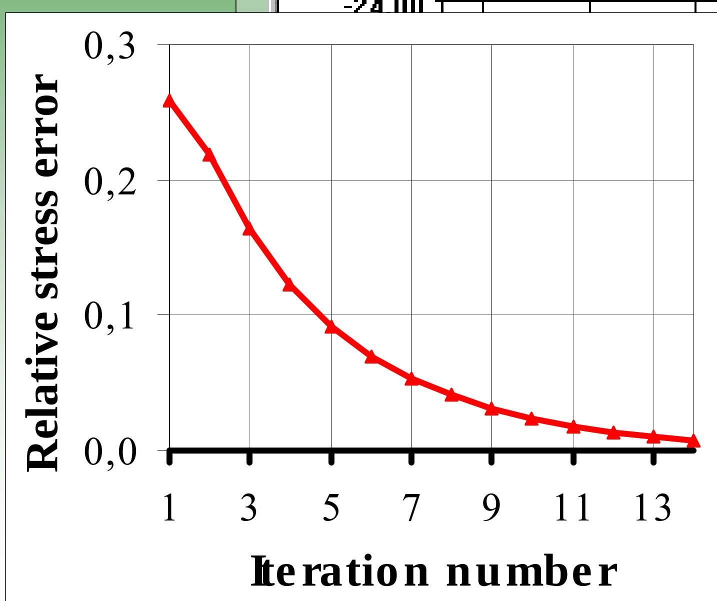


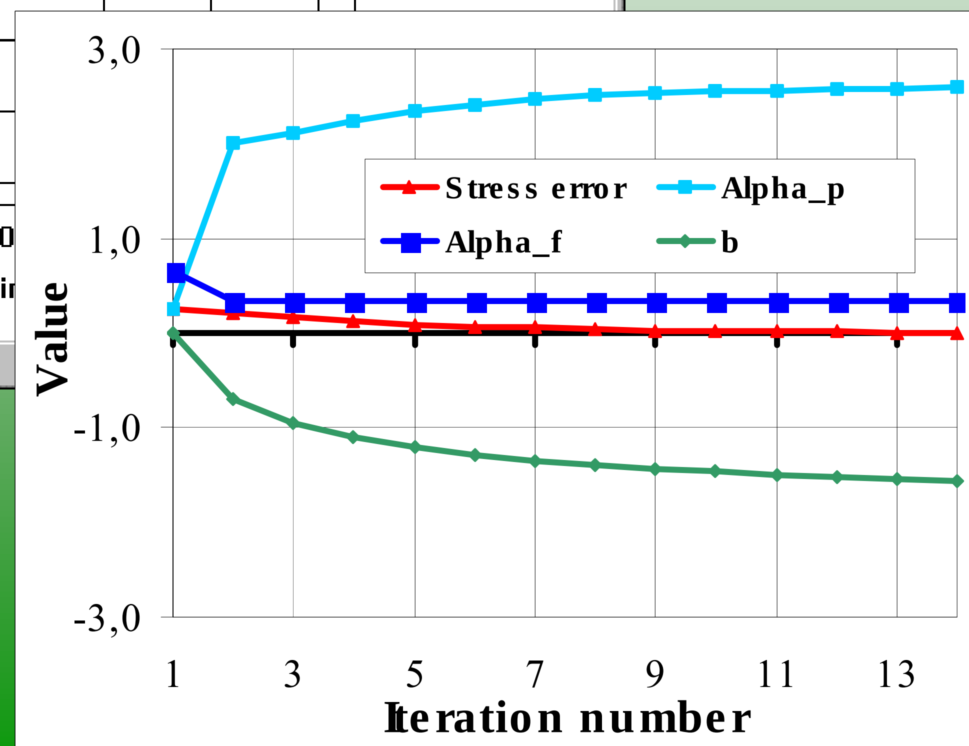
Tri-axial loading in tension and compression

Fracture Plastic Model









Crack closure

Fracture Plastic Model